

RESEARCH MEMORANDUM

ANALYSIS AND CONSTRUCTION OF DESIGN CHARTS FOR TURBINES
WITH DOWNSTREAM STATORS

By Richard H. Cavicchi and Anita B. Constantine

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS
WASHINGTON

November 18, 1954
Declassified January 12, 1961

THE UNIVERSITY OF CHICAGO

PHYSICS

DEPARTMENT OF PHYSICS

PHYSICS

PHYSICS

PHYSICS

PHYSICS

PHYSICS

PHYSICS

PHYSICS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

ANALYSIS AND CONSTRUCTION OF DESIGN CHARTS FOR TURBINES
WITH DOWNSTREAM STATORS

By Richard H. Cavicchi and Anita B. Constantine

SUMMARY

A set of design charts was constructed from an aerodynamic analysis made for one-stage turbines having both one and two rows of downstream stator blades. The action of the downstream staters in turning the flow at the rotor exit to the axial direction permits the use of higher turbine rotor-exit whirl than is used in conventional designs without the high inherent loss in the exhaust nozzle. Greater work capacity can therefore be produced by one-stage turbines with downstream staters than by orthodox one-stage turbines.

In the analysis, high aerodynamic limits are assigned the turbine rotor velocities. The downstream staters are limited by diffusion factors and solidities currently in use in transonic compressor design. In an example illustrating the use of the charts, it is found that a turbine with one downstream stator has 19 percent greater weight-flow capacity than a conventional one-stage turbine of the same work output, turbine stress, and aerodynamic limits. Or, alternatively, a turbine with one downstream stator has 25 percent greater work capacity for the same equivalent weight flow per unit turbine frontal area, turbine stress, and turbine aerodynamic limits.

INTRODUCTION

In the design of turbojet engines, one of the primary decisions that must be made is the selection of the number of turbine stages. Reference 1 presents a rapid method for use in turbine design by which the number of turbine stages can be determined to satisfy the turbine design requirements without exceeding specified turbine aerodynamic limits. This method yields turbine designs of which the exit whirl is zero or low in magnitude. The situation frequently arises, however, that one integral number of turbine stages with low exit whirl, even with high aerodynamic limits, cannot satisfy the turbine design requirements; whereas, such a turbine with one more stage, even with conservative aerodynamic limits, possesses a weight-flow and work capacity well in excess of that required.

This very situation is clearly shown for orthodox one- and two-stage turbines in figure 4(a) of reference 2, which presents engine design-point characteristics obtainable from high-output one-stage turbines and also from conservative two-stage turbines on a single map. Such a map reveals a noticeable gap between the capacities of one- and two-stage turbines designed with low exit whirl. It appears, therefore, that a one-stage turbine with a high magnitude of exit whirl might prove worthy of investigation in the interest of bridging this gap.

In conventional turbine designs for turbojet engines, although a high magnitude of tangential component of turbine-exit velocity yields high turbine specific work, an effort is generally made to keep the magnitude of this component low. The reason for doing so, as shown in reference 3, is that high whirl at the exit of turbines designed for use in turbojet engines results in inefficient expansion in the exhaust nozzle, thus causing high losses in thrust. If a row of stator blades is mounted downstream of the turbine rotor, however, the action of these blades in turning the flow to the axial direction should make it possible to design for higher whirl at the rotor exit than is conventionally used, with the expectation that the majority of the thrust losses usually associated with such high whirl can be avoided.

In order to prevent high velocity from adversely affecting the performance of the engine components downstream of the turbine, diffusion of the turbine exhaust gases is usually provided for. If high exit whirl is used, the absolute velocity at the turbine rotor exit will be high. Therefore, a large amount of diffusion will be required of a downstream stator in addition to a large amount of turning of the flow. Some criterion is needed to relate the amount of diffusion and the associated loss. In reference 4 is developed a blade-loading parameter for axial-flow-compressor blade elements that is shown by experiment to provide a better measure of maximum loading than the more generally used coefficient of lift. This parameter, the "diffusion factor," is an index of the amount of diffusion of the flow that a row of blades accomplishes. The diffusion factor can be calculated from the velocity diagram, since it consists of a term involving the relative velocity ratio across the blade and a term proportional to the circulation about the element. Currently, a diffusion factor of 0.2 is considered conservative, 0.4 is moderate, and 0.6 is critical. For a given value of diffusion factor, two rows of downstream stator blades should provide both greater diffusion and more turning of the flow than one row of downstream stator blades.

This report presents design charts made at the NACA Lewis laboratory for one-stage turbines with both one and two rows of downstream stator blades. In each case, a straight annulus is assumed for the downstream stator. The presentation is similar in format to that of reference 1. The charts of the present report are constructed with high turbine aerodynamic limits; the rotor-inlet relative Mach number is 0.8 and the exit

axial Mach number is 0.7, both at the hub radius. Provision for the inevitable loss in the turbine rotor is made by selecting a value for turbine rotor adiabatic efficiency. To provide for loss in the downstream stators, diffusion factors of 0.2, 0.4, and 0.6 are used for the downstream stator limits. An illustrative example is worked and results read from the charts in order to provide a cursory evaluation of turbines with downstream stators.

ANALYSIS AND PREPARATION OF CHARTS

Assumptions

The symbols used in this report are listed in appendix A. The following assumptions were made in the analysis:

- (1) Simplified radial equilibrium
- (2) Free-vortex velocity distribution
- (3) At the mean radius, $(\rho V_x)_m^A$ equal to integrated value of weight flow over blade height: $\dot{w} = (\rho V_x)_{1,m} A_1 = (\rho V_x)_{2,m} A_2$
- (4) No radial variation in stagnation state relative to stator
- (5) Hub and mean radii constant in value from inlet to outlet of rotor
- (6) Constant annular area across downstream stators
- (7) Solidity of downstream stator blades at mean radius, 1.5
- (8) Constant value of $4/3$ for ratio of specific heats for hot gas
- (9) The allowable amount of exit whirl will be determined by the ability of the downstream stators to turn the flow back to the axial direction

Analytical Basis for Charts and Figures

In constructing the charts presented herein, aerodynamic limits were assigned and the analysis was made at the hub radius of rotor and stators, because turbine flow conditions are most critical at this radius. In using the analysis to develop the charts, ranges of blade speed, hub-tip radius ratio, and entrance whirl component were assigned. The charts are presented to facilitate the determination of turbine work output and the apportioning of the tangential-velocity ratio $-(V_{u,1}/V_{u,2})_h$.

As in reference 1, the work output is presented in the nondimensional form $-gJ\Delta h'/U_t^2$, where the negative sign indicates a drop in

enthalpy. This ratio, referred to as the stage-work parameter, is merely one-half the reciprocal of the square of the blade tip to jet speed ratio.

Another parameter presented in the charts is the isentropic annular-area ratio $(A_2/A_1)_s$, which is the ratio of the annular area behind the rotor to that ahead of the rotor with the assumption of isentropic flow. Equation (C12) of reference 1 defines this parameter as

$$\left(\frac{A_2}{A_1}\right)_s \equiv \frac{A_2}{A_1} \left(\frac{p_2''}{p_1''}\right)_m$$

Figure 1 shows a typical velocity diagram together with the station numbers used throughout this report. One prime superscript designates a thermodynamic property relative to the stator, and two primes, relative to the rotor.

In making the analysis, it was necessary first to establish conditions for the downstream stators, following which the rotor analysis could be made.

Downstream stator analysis. - The main object in using downstream stators with turbines is that high whirl is tolerable at the turbine rotor exit with concomitant higher specific work. Increase of turbine-exit whirl is also accompanied, however, by decrease in turbine-exit specific weight flow. Appendix B gives the details of the procedure used in establishing the conditions for one downstream stator, and appendix C gives those for two downstream stators. For the entire downstream stator analysis, a value of 0.7 is used for $(V_x/a)_{2,h}$.

One downstream stator: From assigned values of turbine hub-tip radius ratio r_h/r_t and values of turbine-exit axial and tangential Mach numbers at the hub radius, $(V_x/a)_{2,h}$ and $(V_u/a)_{2,h}$, respectively, turbine-exit specific weight flow $(\rho V_x/\rho' a'_{cr})_{2,m}$ was determined. The value of this same parameter at the exit from the downstream stator $(\rho V_x/\rho' a'_{cr})_{3,m}$ was next calculated by the relation

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{3,m} = \left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{2,m} \frac{p_2'}{p_3'} \quad (B6)$$

where

$$\frac{p_3'}{p_2'} = 1 - \bar{\omega} \left\{ 1 - \left[1 + \frac{k-1}{2} \left(\frac{V}{a}\right)_{2,h}^2 \right]^{\frac{-k}{k-1}} \right\} \quad (B7)$$

The stagnation-pressure loss coefficient $\bar{\omega}$ is defined in reference 4 as

$$\bar{\omega} \equiv \frac{p'_{3,ideal} - p'_3}{p'_2 - p_2} \quad (1)$$

Because the flow is considered straightened to the axial direction at the downstream stator exit, figure 3 of reference 5 established the stagnation condition at the exit of the downstream stator. The solidity of the downstream stator was calculated for the range of assigned parameters by

$$\sigma_{2,h} = \frac{\frac{1}{2} \frac{|V_{u,2,h}|}{V_{2,h}}}{D_{2,h} - 1 + \left(\frac{V_3}{V_2}\right)_h} \quad (B13)$$

where $D_{2,h}$ is the diffusion factor reported in reference 4. Finally, a preliminary plot of $\sigma_{2,h}$ against $(V_u/a)_{2,h}$ with lines of constant r_h/r_t was made for three constant values of $D_{2,h}$.

Two downstream stators: Values were assigned for turbine hub-tip radius ratio, for $(V_x/a)_{2,h}$, and for diffusion factor and solidity of the two downstream stators. It was necessary to assume a value of $(V_u/a)_{2,h}$ in order to determine $(\rho V_x / \rho' a'_{cr})_{2,m}$, from which

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{3,m} = \left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{2,m} \frac{p'_2}{p'_3} \quad (B6)$$

Several trial values of $(V_x/a'_{cr})_3$ were then assumed in order to calculate diffusion factor for the first downstream stator $D_{2,h}$. From a plot of $D_{2,h}$ against $(V_x/a'_{cr})_3$, the value of the latter could be read at the desired value of $D_{2,h}$. Calculation of stagnation-pressure ratio across the second downstream stator by

$$\frac{p'_4}{p'_3} = 1 - \bar{\omega} \left\{ 1 - \left[1 - \frac{k-1}{k+1} \left(\frac{V}{a'_{cr}}\right)_{3,h}^2 \right]^{\frac{k}{k-1}} \right\} \quad (C1)$$

served to permit evaluation of $(\rho V_x / \rho' a'_{cr})_{4,m}$ by

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{4,m} = \left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{3,m} \frac{p'_3}{p'_4} \quad (C2)$$

With the condition that the velocity at the exit of the second downstream stator is axial,

$$\left(\frac{V_u}{a'_{cr}}\right)_4 = 0 \quad (C3)$$

diffusion factor of the second downstream stator was calculated from

$$D_{3,h} = 1 - \left(\frac{V_x}{a'_{cr}}\right)_4 \left(\frac{a'_{cr}}{V}\right)_{3,h} + \frac{1}{2\sigma_{3,h}} \left(\frac{V_u}{a'_{cr}}\right)_{3,h} \left(\frac{a'_{cr}}{V}\right)_{3,h} \quad (C4)$$

This procedure was repeated for several values of $(V_u/a)_{2,h}$, and a plot of $D_{3,h}$ against $(V_u/a)_{2,h}$ was made. The value of $(V_u/a)_{2,h}$ could then be determined at the desired value of $D_{3,h}$.

Turbine rotor analysis. - The starting point in the analysis of the turbine rotor was the assignment of values of turbine rotor-inlet relative Mach number $(W/a)_{1,h}$, entrance blade hub speed ratio $(U/a'_{cr})_{1,h}$, entrance tangential-velocity ratio $(V_u/a'_{cr})_{1,h}$, exit axial Mach number $(V_x/a)_{2,h}$, and hub-tip radius ratio r_h/r_t . For the selected downstream stator solidity at the mean radius of 1.5, the preliminary plot described in the analysis of one downstream stator yielded a unique value of $(V_u/a)_{2,h}$ for each hub-tip radius ratio. Likewise, the final plot made in the analysis for two downstream stators also yielded a unique value of $(V_u/a)_{2,h}$ for each hub-tip radius ratio.

The calculations then proceeded in the following manner, the details of which are described in appendix D. The principal dependent variables determined are stage-work parameter,

$$\frac{-gJ\Delta h'}{U_t^2} = \frac{\left[\left(\frac{V_u}{a'_{cr}}\right)_{1,h} - \left(\frac{V_u}{a'_{cr}}\right)_{2,h} \sqrt{\frac{T_2'}{T_1'}}\right] \left(\frac{r_h}{r_t}\right)^2}{\left(\frac{U}{a'_{cr}}\right)_{1,h}} \quad (D21)$$

and isentropic annular-area ratio,

$$\left(\frac{A_2}{A_1}\right)_s = \left(\frac{V_x}{a'_{cr}}\right)_1 \left(\frac{a'_{cr}}{V_x}\right)_2 \sqrt{\frac{T_1'}{T_2'}} \left[\frac{(T/T'')_1}{(T/T'')_2}\right]_m^{\frac{1}{k-1}} \quad (D12)$$

In solving equation (D12), use was made of the relation

$$\left(\frac{V_x}{a'_{cr}}\right)_1^2 = \frac{\frac{M_{1,h}^2}{2} \left[k + 1 - (k-1) \left(\frac{V_u}{a'_{cr}}\right)_{1,h}^2 \right] - \left(\frac{V_u}{a'_{cr}} - \frac{U}{a'_{cr}}\right)_{1,h}^2}{1 + \frac{k-1}{2} M_{1,h}^2} \quad (D3)$$

in which $M_{1,h} = (W/a)_{1,h}$. The turbine stagnation-temperature ratio T'_2/T'_1 of equation (D21) was calculated from

$$\sqrt{\frac{T'_2}{T'_1}} = \sqrt{\frac{k+1}{2}} \left(\frac{k-1}{k+1}\right) \left(\frac{U}{a'_{cr}}\right)_{1,h} \left(\frac{V_u}{a}\right)_{2,h} \sqrt{\left(\frac{T}{T'}\right)_{2,h}} \pm \sqrt{1 - \frac{k-1}{k+1} \left(\frac{U}{a'_{cr}}\right)_{1,h} \left[2 \left(\frac{V_u}{a'_{cr}}\right)_{1,h} - \frac{k-1}{2} \left(\frac{U}{a'_{cr}}\right)_{1,h} \left(\frac{V_u}{a}\right)_{2,h} \left(\frac{T}{T'}\right)_{2,h} \right]} \quad (D7)$$

Preliminary plots of stage-work parameter $-gJ\Delta h'/U_t^2$ against isentropic annular-area ratio $(A_2/A_1)_s$ with lines of constant entrance blade hub speed ratio $U_h/a'_{cr,1}$ were constructed, each such plot being drawn for one of the assigned values of hub-tip radius ratio r_h/r_t . Similarly, preliminary plots of $-(V_{u,1}/V_{u,2})_h$ against $(A_2/A_1)_s$ were made. Charts I and III are cross plots of these preliminary plots for isentropic annular-area ratios of 0.8, 0.9, 1.0, and 1.1. Chart I is prepared for turbines with one row of downstream stator blades limited by diffusion factors of 0.2, 0.4, and 0.6. The turbine designs of chart III have two rows of downstream stator blades limited by diffusion factors of 0.2 and 0.4. A diffusion factor of 0.6 is not presented in chart III, because, despite the increase in exit whirl, the specific weight-flow parameter $(\rho V_x/\rho' a'_{cr})_{2,m}$ decreases to such an extent that the weight-flow capacities are less than those of turbines with two rows of downstream stator blades and a diffusion factor of 0.4.

Charts I(a) and III(a), which are used for determining turbine work, consist of stage-work parameter $-gJ\Delta h'/U_t^2$ plotted against blade-speed parameter $U_t/a'_{cr,1}$ with lines of constant hub-tip radius ratio for a constant value of isentropic annular-area ratio $(A_2/A_1)_s$. Charts I(b) and III(b), which are for apportioning tangential velocity, are presented in the same fashion, but with $-(V_{u,1}/V_{u,2})_h$ as ordinate. The abscissas of all the charts were calculated from the entrance blade hub speed ratio by

$$\frac{U_t}{a'_{cr,1}} = \frac{U_h/a'_{cr,1}}{r_h/r_t} \quad (2)$$

In charts I and III are drawn dividing lines, curves to the right of which are dashed. The dividing lines locate turbine designs in which the flow is turned 120° by the turbine rotor, and the dashed lines apply to turbine designs in which the rotor turning angles exceed 120° . Preliminary plots were constructed of rotor turning angle $\Delta\beta_h$ against isentropic annular-area ratio $(A_2/A_1)_s$ with lines of constant entrance blade hub speed ratio $U_h/a'_{cr,1}$, each such plot being drawn for one of the assigned values of hub-tip radius ratio r_h/r_t . The value of $(A_2/A_1)_s$ established in such a plot by the intersection of the 120° -turning-angle line with a line of constant $U_h/a'_{cr,1}$ located a point on the corresponding preliminary plot of $-gJ\Delta h'/U_t^2$ against $(A_2/A_1)_s$ on a line of constant $U_h/a'_{cr,1}$. From such a procedure, a cross plot was made of $-gJ\Delta h'/U_t^2$ against $(A_2/A_1)_s$ with lines of constant r_h/r_t , the entire cross plot thus being drawn for 120° of turning. This cross plot was used to locate the positions of the dividing lines in chart III(a). The dividing lines in chart III(b) were established by this same cross-plotting procedure, but using $-(V_{u,1}/V_{u,2})_h$ in place of $-gJ\Delta h'/U_t^2$.

The 120° limit on rotor relative turning angle was arbitrarily selected as an upper limit for good turbine performance, as was done in reference 1. It was expected that the law of diminishing returns with respect to turbine work output would begin to take effect with the higher turning angles. The presumption is verified in the DISCUSSION section of this report.

Charts II and IV present turbine designs limited by 120° rotor relative turning angle for the corresponding dashed region of charts I and III. In charts II(a) and IV(a), stage-work parameter is plotted against blade-speed parameter with lines of constant hub-tip radius ratio. Charts II(b) and IV(b) are presented in the same manner, but $-(V_{u,1}/V_{u,2})_h$ replaces $-gJ\Delta h'/U_t^2$ as ordinate. Since in making the calculations for charts II and IV an additional constraint was used ($\Delta\beta_h = 120^\circ$), there results a unique value of $(A_2/A_1)_s$ for any given values of $U_h/a'_{cr,1}$ and r_h/r_t . Therefore, for each diffusion factor presented in charts II and IV, there is but one plot. The dotted lines of constant $(A_2/A_1)_s$ in charts II and IV show the amount of divergence in the annulus across the rotor that must be used if the turbine design is to be limited to 120° of turning.

Chart II was prepared for turbine designs with one row of downstream stator blades having diffusion factors of 0.4 and 0.6. A diffusion factor

of 0.2 is not presented in chart II, because the rotor relative turning angle $\Delta\beta_h$ is less than 120° for all but a small region of charts I(a) and (b). Because the rotor relative turning angles of turbine designs having two rows of downstream stator blades can exceed 120° for all three diffusion factors considered, chart IV is made for all three values.

Although the charts were prepared on the basis of isentropic flow, provision is made for including an assumption for turbine rotor adiabatic efficiency. Because the efficiency factor applies to the rotor alone, any determination of the over-all turbine efficiency must take into account the loss assumed for the downstream stators ω . If a factor for turbine rotor adiabatic efficiency is included, the velocity diagram and stage-work parameter obtained from the charts are not affected directly; rather, the effect is realized directly by the annular-area ratio. In reference 1 a loss factor $e^{\frac{J\Delta s}{R}}$ is developed, which is used according to the equation

$$\frac{A_2}{A_1} = \left(\frac{A_2}{A_1} \right)_s e^{\frac{J\Delta s}{R}} \quad (3)$$

This loss factor, which is a function of turbine rotor adiabatic efficiency, is a convenient means of expressing the deviation from isentropic flow. Because the loss factor is expressed in terms of the entropy increase across the rotor, its value is always greater than 1 when the turbine rotor adiabatic efficiency is less than 100 percent.

The charts are constructed for various isentropic annular-area ratios in order that interpolation between the actual annular-area ratios calculated from equation (3) can be used in determining the stage-work parameter and $-(V_{u,1}/V_{u,2})_h$ corresponding to the desired annular-area ratio. Figure 2, which is an extension of figure 4(b) in reference 1, is a plot of loss factor $e^{\frac{J\Delta s}{R}}$ against entrance stage-enthalpy parameter $-gJ\Delta h'/T_1'$ with lines of constant turbine rotor adiabatic efficiency η_T .

The turbine weight flow is expressed in the form of a nondimensional parameter identified in figure 3 as exit weight-flow parameter $\hat{w}_2$. This parameter is calculated from the equation

$$\hat{w}_2 = \left\{ 1 - \frac{k-1}{k+1} \left[\left(\frac{V_u}{a'_{cr}} \right)_{2,m}^2 + \left(\frac{V_x}{a'_{cr}} \right)_2^2 \right] \right\}^{\frac{1}{k-1}} \left(\frac{V_x}{a'_{cr}} \right)_2 \left[1 - \left(\frac{r_h}{r_t} \right)^2 \right] \quad (D23)$$

In figures 3(a) and (b), r_h/r_t is plotted against $\hat{w}_2$ for turbines with one and two rows of downstream stator blades, respectively. Table I presents a summary of the design parameters used in the construction of the charts.

USE OF CHARTS AND ILLUSTRATIVE EXAMPLE

Use of Charts

The turbine design charts presented herein can be used either (1) to determine a turbine design from a set of turbine design requirements or (2) to prepare a whole map of design-point potentialities of one-stage turbines with downstream stators.

Turbine design requirements. - The set of turbine design requirements alluded to in item (1) above consists, in one form or another, of some of the following parameters:

(a) Equivalent work per pound of gas needed to drive compressor, $-\Delta h'/\theta'_{C,i}$

(b) Equivalent weight flow per unit turbine tip frontal area, $w\sqrt{\theta'_{C,i}}/A_F\delta'_{C,i}$

(c) Equivalent blade tip speed of turbine, $U_t/\sqrt{\theta'_{C,i}}$

(d) Equivalent turbine-inlet stagnation temperature relative to stator, $T'_1/\theta'_{C,i}$

(e) Equivalent turbine-inlet stagnation pressure relative to stator, $p'_1/\delta'_{C,i}$

Calculation of parameters. - Certain thermodynamic parameters must first be calculated in order to express the turbine design requirements in the nondimensional form presented in the charts. For this purpose, the following equations are helpful:

$$T'_2 = T'_1 - \left(\frac{-\Delta h'_{req}}{c_p} \right) \quad (4)$$

$$p'_2 = p'_1 \left(1 - \frac{T'_1 - T'_2}{\eta_T T'_1} \right)^{\frac{k}{k-1}} \quad (5)$$

$$\rho_2' = \frac{p_2'}{RT_2'} \quad (D9)$$

$$a_{cr,2}' = \sqrt{\frac{2k}{k+1} gRT_2'} \quad (6)$$

$$\frac{U_t}{a_{cr,1}'} = \frac{U_t}{\sqrt{\theta_{C,i}'}} \sqrt{\frac{T_{C,i}'}{518.7}} \frac{1}{a_{cr,1}'} \quad (7)$$

$$\hat{w}_2 = \frac{w \sqrt{\theta_{C,i}'}}{A_F \delta_{C,i}'} \frac{p_{C,i}'}{2116} \sqrt{\frac{518.7}{T_{C,i}'}} \left(\frac{1}{\rho' a_{cr,1}'} \right)_2 \quad (8)$$

Required stage-work parameter:

$$\left(\frac{-gJ\Delta h'}{U_t^2} \right)_{req} \quad (9)$$

Entrance stage-enthalpy parameter:

$$\frac{-gJ\Delta h'_{req}}{T_1'} \quad (10)$$

Compressor specific work:

$$\Delta h_C' = \frac{T_{C,i}'}{\eta_C} \frac{\gamma}{\gamma-1} \frac{R}{J} \left[\left(\frac{p_{C,o}'}{p_{C,i}'} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad (11)$$

Details of using charts for item (1). - With a given set of turbine design requirements, the initial step is to assign a value for the diffusion factor $D_{2,h}$ of the downstream stator along with turbine rotor annular-area ratio A_2/A_1 and turbine rotor adiabatic efficiency η_T .

As indicated by equation (10), the entrance stage-enthalpy parameter is calculated with the required enthalpy drop. From this value of entrance stage-enthalpy parameter and the assumed turbine rotor adiabatic efficiency, a loss factor can be read on figure 2. The actual annular-area ratio is calculated from this loss factor by

$$\frac{A_2}{A_1} = \left(\frac{A_2}{A_1} \right)_s e^{\frac{J\Delta s}{R}} \quad (3)$$

The procedure is to select two isentropic annular-area ratios so that the resulting values of A_2/A_1 by equation (3) encompass the desired annular-area ratio. A linear interpolation is used to evaluate the particular dependent turbine parameter sought.

The procedure then depends upon the particular turbine design requirements given. The following describes briefly one possible method of using the charts:

Given: work, blade speed, turbine-inlet temperature, and compressor-inlet temperature.

After $-gJ\Delta h'_{\text{req}}/T_1$ is calculated, the loss factor $e^{\frac{J\Delta s}{R}}$ can be read from figure 2.

After the work is expressed as $-gJ\Delta h'/U_t^2$, the value of r_h/r_t is read at the point located by $-gJ\Delta h'/U_t^2$ and $U_t/a'_{\text{cr},1}$ on the charts for two values of $(A_2/A_1)_s$, so that the corresponding two values of A_2/A_1 encompass the desired annular-area ratio A_2/A_1 .

Following a linear interpolation of r_h/r_t to the desired value of A_2/A_1 , figure 3 immediately yields turbine-exit weight-flow parameter $\hat{w}_2$.

At the known value of $U_t/a'_{\text{cr},1}$ and that value determined for r_h/r_t , the ratio $-(V_{u,1}/V_{u,2})_h$ can be read from the charts and interpolated linearly for the desired annular-area ratio. At this point, the turbine design is determined.

The ratio $-(V_{u,1}/V_{u,2})_h$ permits apportioning the amount of whirl at the inlet to and exit from the turbine rotor. In this connection, the following variation of equation (D21) is useful:

$$\left(\frac{V_u}{a'_{\text{cr}}}\right)_{2,h} = \frac{-gJ\Delta h'}{U_t^2} \frac{\frac{U_t}{a'_{\text{cr},1}} \sqrt{\frac{T_1}{T_2}}}{\left[\left(\frac{V_{u,1}}{V_{u,2}}\right)_h - 1\right] \frac{r_h}{r_t}} \quad (12)$$

Turbine-inlet whirl parameter is then

$$\left(\frac{V_u}{a'_{\text{cr}}}\right)_{1,h} = \left(\frac{V_{u,1}}{V_{u,2}}\right)_h \sqrt{\frac{T_2}{T_1}} \left(\frac{V_u}{a'_{\text{cr}}}\right)_{2,h} \quad (13)$$

where

$$\frac{T_2'}{T_1'} = 1 - 2 \frac{k-1}{k+1} \left(\frac{-gJ\Delta h'}{U_t^2} \right) \left(\frac{U_t}{a_{cr,1}'} \right)^2 \quad (14)$$

Details for using charts for item (2). - The present report may perhaps be most useful if used in the manner by which reference 6 was prepared from the charts of reference 1. In this way, a whole map of design-point potentialities of one-stage turbines with downstream stators can be easily determined.

Values can be assigned to the following variables:

$$\begin{array}{ll} r_h/r_t & \eta_T \\ U_t/a_{cr,1}' & \eta_C \\ A_2/A_1 & T_1'/T_{C,1}' \end{array}$$

From these assigned values, the charts can be read to yield $-gJ\Delta h'/U_t^2$ and $\hat{w}_2$.

The compressor pressure ratio can be determined from $-gJ\Delta h'/U_t^2$ by

$$\frac{p_{C,o}'}{p_{C,i}'} = \left[1 + \frac{2k}{k+1} \frac{\gamma-1}{\gamma} \eta_C \frac{T_1'}{T_{C,i}'} \left(\frac{U_t}{a_{cr,1}'} \right)^2 \frac{-gJ\Delta h'}{U_t^2} \right]^{\frac{\gamma}{\gamma-1}} \quad (15)$$

The equivalent weight flow per unit turbine frontal area is calculated from $\hat{w}_2$ by

$$\frac{w\sqrt{\theta_{C,i}'}}{A_F\delta_{C,i}'} = \frac{2116}{\sqrt{518.7}} \sqrt{\frac{2k}{k+1} \frac{g}{R}} \sqrt{\left(\frac{T_{C,i}'}{T_1'} \right) \frac{T_1'}{T_2'}} \cdot \frac{p_2'}{p_1'} \cdot \frac{p_1'}{p_{C,o}'} \cdot \frac{p_{C,o}'}{p_{C,i}'} \hat{w}_2 \quad (16)$$

in which equation (14) is used for evaluating T_2'/T_1' , equation (5) for p_2'/p_1' , and equation (15) for $p_{C,o}'/p_{C,i}'$.

Illustrative Example

The following example illustrates the manner of using the charts to determine a turbine design from a set of requirements:

Turbine design requirements

Assigned variables

$$p'_{C,o}/p'_{C,i} = 4.0$$

$$M_0 = 2.8 \text{ in stratosphere} \quad \bar{\omega} = 0.05$$

$$U_t/\sqrt{\theta'_{C,i}} = 930 \text{ ft/sec}$$

$$A_2/A_1 = 1.0$$

$$\eta_C = 0.85$$

$$T'_1 = 3004^\circ \text{ R}$$

$$M_{1,h} = 0.8$$

$$\eta_T = 0.85$$

$$(V_x/a)_{2,h} = 0.7$$

$$k = 4/3$$

$$D_{2,h} = 0.4$$

$$p'_1/p'_{C,o} = 0.95$$

For $M_0 = 2.8$ in the stratosphere, $\theta'_{C,i} = 1001/518.7 = 1.930$. Also,

$c_p = \frac{k}{k-1} \frac{R}{J} = 0.2745 \text{ Btu}/(\text{lb})(^\circ\text{R})$. In this example, one row of downstream stator blades is assumed.

Calculation of parameters. -

$$\Delta h'_c = \frac{1001}{0.85} (0.240)(4^{1/3.5} - 1) = 137.4 \text{ Btu/lb} \quad \text{by eq. (11)}$$

$$U_t = 930\sqrt{1.930} = 1292 \text{ ft/sec} \quad \text{by eq. (7)}$$

$$-gJ\Delta h'/U_t^2 = (25,000)(137.4)/(1292)^2 = 2.058$$

$$a'_{cr,1} = 44.3\sqrt{3004} = 2428 \text{ ft/sec}$$

$$T'_2 = 3004 - 137.4/0.2745 = 2503^\circ \text{ R} \quad \text{by eq. (4)}$$

$$a'_{cr,2} = 44.3\sqrt{2503} = 2216 \text{ ft/sec} \quad \text{by eq. (6)}$$

$$U_t/a'_{cr,1} = 1292/2428 = 0.532$$

$$-gJ\Delta h'/T'_1 = (25,000)(137.4)/3004 = 1143 \text{ sq ft}/(\text{sec}^2)(^\circ\text{R})$$

$$\frac{p'_2}{p'_1} = \left[1 - \frac{3004 - 2503}{0.85(3004)} \right]^4 = 0.4174 \quad \text{by eq. (5)}$$

$$\rho'_2 = \frac{0.4174 p'_1}{53.4(2503)} = 3.123 \times 10^{-6} p'_1 \text{ lb/cu ft} \quad \text{by eq. (D9)}$$

From figure 2 at $-gJ\Delta h'/T'_1 = 1143 \text{ sq ft}/(\text{sec}^2)(^\circ\text{R})$ and $\eta_T = 0.85$,
 $\frac{J\Delta s}{e R} = 1.153$.

Entry into charts. - From chart I(a)2 at $-gJ\Delta h'/U_t^2 = 2.058$ and $U_t/a'_{cr,1} = 0.532$,

$(A_2/A_1)_s$	A_2/A_1	r_h/r_t
0.8	0.922	0.724
.9	1.038	.734

Interpolated linearly for a value of 1.0 for A_2/A_1 , the hub-tip radius ratio $r_h/r_t = 0.731$. Likewise, from chart I(b)2 at $U_t/a'_{cr,1} = 0.532$ and $r_h/r_t = 0.731$,

$(A_2/A_1)_s$	A_2/A_1	$-(V_{u,1}/V_{u,2})_h$
0.8	0.922	2.524
.9	1.038	2.465

Interpolated linearly for a value of 1.0 for A_2/A_1 , the ratio $-(V_{u,1}/V_{u,2})_h = 2.484$. At $r_h/r_t = 0.731$, figure 3(a) yields $\hat{w}_2 = 0.2465$.

Calculation of equivalent weight flow per unit turbine frontal area, stress, and over-all efficiency. - The equivalent weight flow per unit turbine frontal area is (by eq. (8)):

$$\begin{aligned}
 \frac{w\sqrt{\theta'_{C,i}}}{A_F\delta'_{C,i}} &= \frac{2116}{P'_{C,i}} \rho'_2 \sqrt{\theta'_{C,i}} a'_{cr,2} \hat{w}_2 \\
 &= 2116(3.123 \times 10^{-6}) \frac{P'_1}{P'_{C,o}} \frac{P'_{C,o}}{P'_{C,i}} \sqrt{\theta'_{C,i}} a'_{cr,2} \hat{w}_2 \\
 &= 2116(3.123 \times 10^{-6})(0.95)(4) \sqrt{1.930} (2216)(0.2465) \\
 &= 19.1 \text{ lb}/(\text{sec})(\text{sq ft})
 \end{aligned}$$

For comparison, the centrifugal stress at the hub radius of the turbine rotor blades is calculated. The stress (ref. 7) is

$$S = \frac{\Gamma U_t^2 \psi}{2g(144)} \left[1 - \left(\frac{r_h}{r_t} \right)^2 \right]$$

If the blade density Γ is 500 lb/cu ft and the stress-correction factor for tapered blades ψ is 0.7,

$$s = \frac{500(1292)^2(0.7)}{288(32.17)} [1 - (0.731)^2] = 29,400 \text{ psi}$$

The efficiency η_T assumed in this example is the rotor adiabatic efficiency. This example is terminated with a calculation of over-all adiabatic efficiency. A value of 0.98 is assumed for the stagnation pressure ratio $p'_1/p'_{B,o}$ across the stator upstream of the turbine rotor. By equation (12),

$$\left(\frac{V_u}{a'_{cr}}\right)_{2,h} = 2.058 \frac{0.532 \sqrt{\frac{3004}{2503}}}{(-2.484 - 1)(0.731)} = -0.4710$$

From the energy equation,

$$\left(\frac{T}{T'}\right)_{2,h} = \frac{1 - \frac{k-1}{k+1} \left(\frac{V_u}{a'_{cr}}\right)_{2,h}^2}{1 + \frac{k-1}{2} \left(\frac{V_x}{a}\right)_{2,h}^2} = \frac{1 - \frac{(0.4710)^2}{7}}{1 + \frac{(0.7)^2}{6}} = 0.8952$$

Stagnation-pressure ratio across the downstream stator is calculated by equation (B7):

$$\frac{p'_3}{p'_2} = 1 - 0.05 [1 - (0.8952)^4] = 0.9821$$

Stage stagnation-pressure ratio is

$$\frac{p'_3}{p'_{B,o}} = \frac{p'_3}{p'_2} \frac{p'_2}{p'_1} \frac{p'_1}{p'_{B,o}} = 0.9821(0.4174)(0.98) = 0.4017$$

The over-all adiabatic efficiency of the turbine is then

$$\eta = \frac{-\Delta h'}{\frac{k}{k-1} \frac{R}{J} T'_1 \left[1 - \left(\frac{p'_3}{p'_{B,o}} \right)^{\frac{k-1}{k}} \right]} = \frac{137.4}{0.2745(3004) [1 - (0.4017)^{0.25}]} = 0.817$$

DISCUSSION

Comparison of Turbine Capacities

In order to compare design performances of one-stage turbines having downstream stators with conventional one- and two-stage turbines as well

as to compare various types of turbine designs with downstream stators, the following two tables were prepared from the data presented in the charts. The bases for comparison in the first table are constant values of engine temperature ratio of 3.0, compressor pressure ratio $p'_{C,o}/p'_{C,i}$ of 4.0, and turbine blade centrifugal stress of 30,000 psi. The weight-flow capacities of one-stage turbines with both one and two rows of downstream stator blades (referred to in the tables as $1\frac{1}{2}$ -stage turbines) for all three diffusion factors considered herein can then be compared easily both among themselves as well as with conventional one- and two-stage turbines. The rotor hub turning angle is limited to 120° .

Turbine stages	Rows of downstream stator blades	Diffusion factor, D	Max. hub Mach number, $(W/a)_{1,h}$	$w\sqrt{\theta'_{C,i}/A_F\delta'_{C,i}}$, lb/(sec)(sq ft)
1	0		0.8	16.1
$1\frac{1}{2}$	1	0.2	0.8	18.0
		.4	↓	19.2
		.6	↓	19.8
	2	0.2	0.8	19.2
		.4	↓	19.4
		.6	↓	17.2
2	0		0.6	22.2
			.8	30.7

The weight-flow capacity of the one-stage turbine with one row of downstream stator blades increases as the diffusion factor increases. In the case of two downstream stators, however, the weight-flow capacity increases as diffusion factor increases in value from 0.2 to 0.4 and then decreases for the diffusion factor of 0.6. This occurrence is brought about despite the increase in exit whirl, because, as can be seen in figure 3 of reference 5, as the whirl increases along a line of constant exit axial Mach number $(V_x/a)_{2,h}$, turbine-exit weight-flow parameter $(\rho V_x/\rho' a'_{cr})_{2,m}$ decreases. Data used in the preparation of references 2 and 6 provided the results tabulated for the conventional one- and two-stage turbines. The preceding table shows that a one-stage turbine with one row of downstream stator blades of 0.4 diffusion factor has 19 percent greater weight-flow capacity than a conventional one-stage turbine having the same aerodynamic limits and stress. The one-stage turbine with the downstream stator has both lower blade speed and hub-tip radius ratio.

In the following table, the bases for comparison are constant values of engine temperature ratio of 3.0, equivalent weight flow per unit

turbine frontal area of 16.0 pounds per second per square foot, and turbine blade centrifugal stress of 30,000 psi. Again, the rotor hub turning angle is limited to 120° . The maximum hub Mach number is 0.8 for all designs. This table was prepared for the purpose of comparing obtainable compressor pressure ratios and work outputs for the various types of turbine design considered herein, all designed for the same weight-flow capacity. No results are tabulated for two-stage turbines in this table, because, for such a low weight-flow capacity as 16.0 pounds per second per square foot, the blade speed would be exceedingly low for the two-stage turbines.

Turbine stages	Rows of downstream stator blades	Diffusion factor, D	$\frac{p'_{C,o}}{p'_{C,i}}$	Increase in work capacity over conventional one-stage turbine, percent
1	0		4.05	0
$1\frac{1}{2}$	1	0.2	5.00	19
		.4	5.37	25
		.6	5.53	28
	2	0.2	5.29	24
		.4	5.42	26
		.6	4.54	10

The table shows that obtainable compressor pressure ratio increases with an increase in diffusion factor for one-stage turbines having one row of downstream stator blades. For two downstream-stators, the obtainable compressor pressure ratio increases as diffusion factor is increased from 0.2 to 0.4 and then decreases markedly as the diffusion factor is raised to 0.6. These data show that a one-stage turbine with one row of downstream stator blades of 0.4 diffusion factor has 25 percent greater work capacity than a conventional one-stage turbine having the same aerodynamic limits and stress. The blade speeds and hub-tip radius ratios of both these designs are nearly the same.

Comparison of Turning by One and Two Rows of Downstream Stator Blades

A comparison of stator turning angles for a given diffusion factor, mean-radius solidity, and hub-tip radius ratio showed that the second row of downstream stator blades turns the flow a few more degrees than the one stator alone. Furthermore, the first stator of two rows does but a small amount of turning. In one specific case examined, one row of downstream stator blades turned the flow 37° in straightening the flow. For the same desired variables, the first of two rows of downstream stator blades turned the flow 13° , and the second row turned the flow 39° .

Gain Obtainable by Exceeding 120° Rotor Relative Turning Angle

In the regions on charts I and III for which the rotor relative turning angle exceeds 120° (dashed lines), either high rotor turning angles must be accepted or high divergence of annular area across the rotor must be resorted to (charts II and IV). Therefore, in making such designs a decision must be made as to the easier course to follow. A few points were read from the charts to indicate the increase in turbine work obtainable from designs in which rotor relative turning angle exceeds 120° over the work obtainable from 120° designs. Following is a tabulation of some of the most extreme of these points for an entrance blade-speed parameter of 0.8 (the values of $\Delta\beta_h$ were taken directly from the data used in construction of the charts):

r_h/r_t	D	$(A_2/A_1)_s = 0.9$		Rotor relative turning angle, $\Delta\beta_h = 120^\circ$		Percent difference in $-gJ\Delta h'/U_t^2$
		$-gJ\Delta h'/U_t^2$	$\Delta\beta_h$, deg	$-gJ\Delta h'/U_t^2$	$(A_2/A_1)_s$	
Chart I(a)2				Chart II(a)1		
0.6	0.4	1.228	123	1.215	1.0	1.1
.7		1.462	126	1.440	1.1	1.5
.8		1.711	129	1.670	1.2	2.5
.9		1.971	132	1.910	1.4	3.2
Chart I(a)3				Chart II(a)2		
0.5	0.6	1.145	131	1.110	1.3	3.2
.6		1.378	133	1.330	1.5	3.6
.7		1.622	136	1.558	1.6	4.1
.8		1.873	138	1.790	1.7	4.6
.9		2.138	140	2.035	1.9	5.1
Chart III(a)2				Chart IV(a)2		
0.5	0.4	1.200	135	1.150	1.5	4.3
.6		1.440	137	1.375	1.7	4.7
.7		1.685	139	1.610	1.8	4.7
.8		1.940	141	1.845	2.0	5.1
.9		2.210	143	2.095	2.2	5.5

This tabulation shows that for this investigation the greatest increase in turbine work that can be obtained from designing for high turning angles instead of maintaining a 120° limit is about 5 percent. In the instance shown for one row of downstream stator blades and a 0.6 diffusion factor, the rotor relative turning angle would become 140° to obtain this 5-percent increase. Alternatively, in order to maintain a

120° limit on rotor relative turning angle, with 5 percent less work than is obtainable with 140° of turning, an isentropic annular-area ratio of 1.9 must be employed. For two rows of downstream stator blades with 0.4 diffusion factor, the turbine design must have either 143° of turning through the rotor or an isentropic annular-area ratio of 2.2. A 5-percent increase in turbine work would, for example, increase obtainable compressor pressure ratio from 4.0 to 4.23.

Linearity of Interpolation

Values of parameters read from the charts at isentropic annular-area ratios of 0.8 and 1.0 were interpolated linearly to an isentropic annular-area ratio of 0.9. These interpolated values varied by less than 1/2 percent from the corresponding values read from the charts of 0.9 isentropic annular-area ratio. The usual procedure in using the charts, however, is to interpolate between isentropic annular-area ratios differing by 0.1 rather than by the 0.2 (from 0.8 to 1.0) as was done in this check. It may therefore be concluded that a linear interpolation is sufficiently accurate.

CONCLUDING REMARKS

In an attempt to bridge the gap between the capacities of one- and two-stage turbines, both having low exit whirl, design charts were constructed for high Mach number one-stage turbines having one and two rows of downstream stator blades. With the use of downstream stators, higher whirl can be tolerated at the turbine rotor exit without the associated inherent loss in the exhaust nozzle because of the straightening action of the downstream stators.

For a given value of turbine-exit axial Mach number at the hub radius $(V_x/a)_{2,h}$, the value of turbine-exit specific weight flow $(\rho V_x / \rho' a'_{cr})_{2,m}$ decreases with increase in exit whirl. However, the effect of the increase in turbine work offsets the lowering of $(\rho V_x / \rho' a'_{cr})_{2,m}$ for diffusion factors of 0.2, 0.4, and 0.6 for turbines with one row of downstream stator blades and for diffusion factors of 0.2 and 0.4 for two rows of stators at the same turbine blade centrifugal stress. For two rows of downstream stator blades of 0.6 diffusion factor, the decrease in $(\rho V_x / \rho' a'_{cr})_{2,m}$ occasioned by the high magnitude of exit whirl, for constant $(V_x/a)_{2,h}$, is sufficiently great to reduce the weight-flow capacity of the machine despite the high work capacity. An example was worked in which a turbine with one row of downstream stator blades of 0.4 diffusion factor has 19 percent greater weight-flow capacity than a conventional one-stage turbine of the same engine temperature ratio, work output, aerodynamic limits, and turbine blade centrifugal stress.

Or, alternatively, for the same weight-flow capacity and turbine stress, the turbine having one row of downstream stator blades has 25 percent greater work output than the one-stage turbine. Turbines with downstream stators, therefore, offer promise for use in turbojet-engine design.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, July 28, 1954

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	annular area, sq ft
A_F	turbine tip frontal area, sq ft
a	sonic velocity, $\sqrt{kgRT}$, ft/sec
a'_{cr}	$\sqrt{\frac{2k}{k+1}} gRT'$, ft/sec.
c_p	specific heat at constant pressure
D	diffusion factor, $1 - \frac{V_3}{V_2} + \frac{V_{u,3} - V_{u,2}}{2\sigma V_2}$
g	acceleration due to gravity, 32.17 ft/sec ²
h	specific enthalpy, Btu/lb
J	mechanical equivalent of heat, 778.2 ft-lb/Btu
k	ratio of specific heats for hot gas, 4/3
M	relative Mach number, W/a
p	absolute pressure, lb/sq ft
R	gas constant, 53.4 ft-lb/(lb)(°R)
r	radius, ft
$\bar{r}$	$r_h/r_m = 2(r_h/r_t)/(1 + r_h/r_t)$
S	turbine blade centrifugal stress, psi
s	specific entropy, Btu/(lb)(°R)
T	absolute temperature, °R
U	blade velocity, ft/sec

V	absolute velocity of gas, ft/sec
W	relative velocity of gas, ft/sec
w	weight flow of gas, lb/sec
$\hat{w}$	weight-flow parameter, $w/A_F \rho' a'_{cr}$
β	flow angle of relative velocity measured from tangential direction (fig. 1), deg
Γ	density of turbine blade metal, lb/cu ft
γ	ratio of specific heats for air, 1.40
δ	ratio of pressure to NACA standard sea-level pressure, $p/2116$
η	adiabatic efficiency
θ	ratio of temperature to NACA standard sea-level temperature, $T/518.7$
ρ	density of gas, lb/cu ft
σ	solidity, ratio of aerodynamic chord to pitch
ψ	stress-correction factor for tapered blades
$\bar{\omega}$	stagnation-pressure loss coefficient of downstream stator blades

Subscripts:

B	combustor
C	compressor
h	hub radius
i	inlet
m	mean radius
o	outlet
req	required
s	isentropic
T	turbine

t	tip
u	tangential component
x	axial component
0	free stream
1	station upstream of turbine rotor
2	station downstream of turbine rotor
3	station downstream of first downstream stator
4	station downstream of second downstream stator

Superscripts:

'	stagnation state relative to stator
"	stagnation state relative to rotor

APPENDIX B

DERIVATION OF EQUATIONS FOR USE IN ANALYSIS OF ONE DOWNSTREAM STATOR

Turbine-exit tangential and axial Mach numbers can be written in terms of the turbine-exit critical velocity $a'_{cr,2}$ as follows:

$$\left(\frac{V_u}{a'_{cr}}\right)_{2,m} = \left(\frac{V_u}{a}\right)_{2,h} \bar{r} \sqrt{\frac{k+1}{2} \left(\frac{T}{T'}\right)_{2,h}} \quad (B1)$$

where

$$\bar{r} = \frac{r_h}{r_m} = \frac{2 \frac{r_h}{r_t}}{1 + \frac{r_h}{r_t}} \quad (B2)$$

Likewise,

$$\left(\frac{V_x}{a'_{cr}}\right)_{2,m} = \left(\frac{V_x}{a'_{cr}}\right)_{2,h} = \left(\frac{V_x}{a}\right)_{2,h} \sqrt{\frac{k+1}{2} \left(\frac{T}{T'}\right)_{2,h}} \quad (B3)$$

The temperature ratio $(T/T')_{2,h}$ is evaluated by

$$\left(\frac{T}{T'}\right)_{2,h} = 1 + \frac{k-1}{2} \left[\left(\frac{V_u}{a}\right)_{2,h}^2 + \left(\frac{V_x}{a}\right)_{2,h}^2 \right] \quad (B4)$$

Turbine-exit specific weight flow is calculated from

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{2,m} = \left[1 - \frac{k-1}{k+1} \left(\frac{V_u}{a'_{cr}}\right)_{2,m}^2 - \frac{k-1}{k+1} \left(\frac{V_x}{a'_{cr}}\right)_{2,m}^2 \right]^{\frac{1}{k-1}} \left(\frac{V_x}{a'_{cr}}\right)_{2,m} \quad (B5)$$

From continuity (assuming a straight annulus for the downstream stator),

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{3,m} = \left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{2,m} \frac{p_2^1}{p_3^1} \quad (B6)$$

The stagnation-pressure ratio across the downstream stator can be determined for a given stagnation-pressure loss coefficient $\bar{\omega}$ by the following equation, which is taken from reference 4 (eq. (B5)):

$$\frac{p_3}{p_2} = 1 - \bar{\omega} \left\{ 1 - \left[1 + \frac{k-1}{2} \left(\frac{V}{a} \right)_{2,h}^2 \right]^{\frac{-k}{k-1}} \right\} \quad (B7)$$

where

$$\left(\frac{V}{a} \right)_{2,h}^2 = \left(\frac{V_u}{a} \right)_{2,h}^2 + \left(\frac{V_x}{a} \right)_{2,h}^2 \quad (B8)$$

Since the function of the downstream stator is to straighten the flow to the axial direction,

$$\left(\frac{V_u}{a_{cr}} \right)_{3,m} = 0 \quad (B9)$$

The axial-velocity ratio $(V_x/a'_{cr})_3$ can easily be determined by means of figure 3 of reference 5 and equations (B6) and (B9). The particular figure cited is merely a graphical solution of equation (B5). The axial-velocity ratio $(V_x/a'_{cr})_3$ can be written as an axial Mach number by the relation

$$\left(\frac{V_x}{a} \right)_{3,h} = \left(\frac{V_x}{a'_{cr}} \right)_3 \sqrt{\frac{2}{k+1} \left(\frac{T'}{T} \right)_{3,h}} \quad (B10)$$

where

$$\left(\frac{T'}{T} \right)_{3,h} = \left[1 - \frac{k-1}{k+1} \left(\frac{V_x}{a'_{cr}} \right)_3^2 \right]^{-1} \quad (B11)$$

In reference 4 the following equation is given for diffusion factor:

$$D_{2,h} = 1 - \left(\frac{V_3}{V_2} \right)_h + \frac{|V_{u,2,h}|}{2\sigma_h V_{2,h}} \quad (B12)$$

which can be solved for downstream stator solidity

$$\sigma_{2,h} = \frac{\frac{1}{2} \frac{|V_{u,2,h}|}{V_{2,h}}}{D_{2,h} - 1 + \left(\frac{V_3}{V_2} \right)_h} \quad (B13)$$

The ratio

$$\frac{|v_{u,2,h}|}{v_{2,h}} = \frac{\left| \left(\frac{v_u}{a} \right)_{2,h} \right|}{\left(\frac{v}{a} \right)_{2,h}}$$

and

$$\left(\frac{v_3}{v_2} \right)_h = \frac{\left(\frac{v}{a} \right)_{3,h} \sqrt{\left(\frac{T}{T'} \right)_{3,h} \left(\frac{T'}{T} \right)_{2,h}}}{\left(\frac{v}{a} \right)_{2,h}}$$

for which equations (B4) and (B11) furnish the necessary temperature ratios. Throughout this analysis

$$\left(\frac{v_x}{a} \right)_{2,h} = 0.7 \quad (B14)$$

APPENDIX C

DERIVATION OF EQUATIONS FOR USE IN ANALYSIS OF TWO DOWNSTREAM STATORS

In order that conditions be matched for the two downstream statoms, a trial-and-error procedure was necessary. Once again,

$$\left(\frac{V_x}{a}\right)_{2,h} = 0.7 \quad (B14)$$

For a given turbine hub-tip radius ratio, a trial value of $(V_u/a)_{2,h}$ is selected. This value, together with equations (B1) to (B5), permits calculation of the turbine-exit specific weight-flow parameter $(\rho V_x/\rho' a'_{cr})_{2,m}$. The value of this parameter at the exit from the first downstream stator is calculated by

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{3,m} = \left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{2,m} \frac{p_2'}{p_3'} \quad (B6)$$

for which the pressure ratio p_2'/p_3' is evaluated by equations (B7) and (B8). At this point several trial values of $(V_x/a'_{cr})_3$ are assumed which, together with $(\rho V_x/\rho' a'_{cr})_{3,m}$, permit determination of $(V_u/a'_{cr})_{3,m}$ by figure 3 of reference 5. Diffusion factor $D_{2,h}$, calculated from its definition given in appendix A, can then be plotted against the several trial values of $(V_x/a'_{cr})_3$ and the value of the latter parameter read at the desired value of $D_{2,h}$. Then the stagnation-pressure ratio across the second downstream stator can be computed by

$$\frac{p_4'}{p_3'} = 1 - \bar{\omega} \left\{ 1 - \left[1 - \frac{k-1}{k+1} \left(\frac{V}{a'_{cr}}\right)_{3,h}^2 \right]^{\frac{k}{k-1}} \right\} \quad (C1)$$

The specific weight-flow parameter at the exit of the second downstream stator

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{4,m} = \left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{3,m} \frac{p_3'}{p_4'} \quad (C2)$$

and the postulated condition that the velocity at the exit of the second downstream stator is axial, that is,

$$\left(\frac{V_u}{a'_{cr}}\right)_4 = 0 \quad (C3)$$

suffice to determine $(V_x/a'_{cr})_4$ on figure 3 of reference 5. Diffusion factor at the inlet to the second downstream stator is

$$D_{3,h} = 1 - \left(\frac{V_x}{a'_{cr}} \right)_4 \left(\frac{a'_{cr}}{V} \right)_{3,h} + \frac{1}{2\sigma_{3,h}} \left(\frac{V_u}{a'_{cr}} \right)_{3,h} \left(\frac{a'_{cr}}{V} \right)_{3,h} \quad (C4)$$

The entire procedure must then be repeated for various assumed values of $(V_u/a)_{2,h}$ and the resulting values of $D_{3,h}$ plotted against $(V_u/a)_{2,h}$. The value of $(V_u/a)_{2,h}$ can be read from the plot at the desired value of $D_{3,h}$. This value of $(V_u/a)_{2,h}$ should then be used to repeat one cycle of calculations in order that the intervening thermodynamic conditions can be evaluated.

APPENDIX D

DERIVATION OF EQUATIONS FOR TURBINE ROTOR ANALYSIS

From the velocity triangle of figure 1,

$$v_{x,1}^2 = w_1^2 - (v_{u,1} - U)^2$$

which can be written

$$\left(\frac{v_x}{a'_{cr}}\right)_1^2 = \frac{k+1}{2} \left(\frac{T}{T'}\right)_{1,h} M_{1,h}^2 - \left(\frac{v_u}{a'_{cr}} - \frac{U}{a'_{cr}}\right)_{1,h}^2 \quad (D1)$$

with

$$\left(\frac{T}{T'}\right)_{1,h} = 1 - \frac{k-1}{k+1} \left[\left(\frac{v_u}{a'_{cr}}\right)_{1,h}^2 + \left(\frac{v_x}{a'_{cr}}\right)_1^2 \right] \quad (D2)$$

and the definitions of a and a'_{cr} . Equations (D1) and (D2) combine to yield

$$\left(\frac{v_x}{a'_{cr}}\right)_1^2 = \frac{M_{1,h}^2}{2} \frac{\left[k + 1 - (k-1) \left(\frac{v_u}{a'_{cr}}\right)_{1,h}^2 \right] - \left(\frac{v_u}{a'_{cr}} - \frac{U}{a'_{cr}}\right)_{1,h}^2}{1 + \frac{k-1}{2} M_{1,h}^2} \quad (D3)$$

In the material that follows, assumption (5) is used, that is,

$$\left. \begin{aligned} r_{1,h} &= r_{2,h} \\ r_{1,m} &= r_{2,m} \end{aligned} \right\} \quad (D4)$$

The work equation is

$$-\Delta h' = \frac{k}{k-1} \frac{R}{J} T'_1 \left(1 - \frac{T'_2}{T'_1} \right) = \frac{U(v_{u,1} - v_{u,2})}{gJ} \quad (D5)$$

from which

$$\frac{T'_2}{T'_1} = 1 - 2 \frac{k-1}{k+1} \left(\frac{U}{a'_{cr}}\right)_{1,h} \left[\left(\frac{v_u}{a'_{cr}}\right)_{1,h} - \left(\frac{v_u}{a}\right)_{2,h} \sqrt{\frac{k+1}{2}} \sqrt{\left(\frac{T}{T'}\right)_{2,h} \frac{T'_2}{T'_1}} \right] \quad (D6)$$

where the ratio $(T/T')_{2,h}$ is given by equation (B4). Solution of equation (D6) for the square root of the turbine stagnation-temperature ratio $\sqrt{T'_2/T'_1}$ yields

$$\sqrt{\frac{T'_2}{T'_1}} = \sqrt{\frac{k+1}{2} \left(\frac{k-1}{k+1} \right) \left(\frac{U}{a'_{cr}} \right)_{1,h} \left(\frac{V_u}{a} \right)_{2,h} \sqrt{\left(\frac{T}{T'} \right)_{2,h}} \pm \sqrt{1 - \frac{k-1}{k+1} \left(\frac{U}{a'_{cr}} \right)_{1,h} \left[2 \left(\frac{V_u}{a'_{cr}} \right)_{1,h} - \frac{k-1}{2} \left(\frac{U}{a'_{cr}} \right)_{1,h} \left(\frac{V_u}{a} \right)_{2,h}^2 \left(\frac{T}{T'} \right)_{2,h} \right]} \quad (D7)$$

In the calculations, the positive sign was used before the radical in equation (D7) in order to yield positive values of $\sqrt{T'_2/T'_1}$. The ratio $(T/T')_{2,h}$ is determined from equation (B4).

By assumption (3), the continuity relation is

$$\rho_{2,m} V_{x,2} A_2 = \rho_{1,m} V_{x,1} A_1 \quad (D8)$$

which, with the use of the equation of state

$$p = \rho RT \quad (D9)$$

becomes

$$\left(\frac{\rho}{\rho''} \right)_{2,m} \frac{p_{2,m}''}{RT_{2,m}''} V_{x,2} A_2 = \left(\frac{\rho}{\rho''} \right)_{1,m} \frac{p_{1,m}''}{RT_{1,m}''} V_{x,1} A_1$$

Relative to the rotor blading, $T_{2,m}'' = T_{1,m}''$; therefore,

$$\frac{A_2}{A_1} \left(\frac{p_2''}{p_1''} \right)_m = \frac{V_{x,1}}{V_{x,2}} \frac{(\rho/\rho'')_{1,m}}{(\rho/\rho'')_{2,m}} \quad (D10)$$

The left side of equation (D10) is defined as the isentropic annular-area ratio $(A_2/A_1)_s$; because, for constant entropy $(p_2''/p_1'')_m = 1$:

$$\left(\frac{A_2}{A_1} \right)_s \equiv \frac{A_2}{A_1} \left(\frac{p_2''}{p_1''} \right)_m \quad (D11)$$

By equation (D11) and the isentropic relation

$$T^{\frac{-1}{k-1}} \rho = \text{constant}$$

equation (D10) becomes

$$\left(\frac{A_2}{A_1}\right)_s = \left(\frac{V_x}{a'_{cr}}\right)_1 \left(\frac{a'_{cr}}{V_x}\right)_2 \sqrt{\frac{T'_1}{T'_2}} \left[\frac{(T/T'')_1}{(T/T'')_2}\right]_m^{\frac{1}{k-1}} \quad (D12)$$

The temperature ratios T/T'' in equation (D12) are evaluated by

$$\frac{T}{T''} = \frac{T}{T'} \frac{T'}{T''} \quad (D13)$$

applied at stations 1 and 2. The relations used to express the temperature ratios of equation (D13) at the mean radius are as follows:

$$\left(\frac{V_u}{a'_{cr}}\right)_{2,h} = \left(\frac{V_u}{a}\right)_{2,h} \sqrt{\frac{k+1}{2}} \sqrt{\left(\frac{T}{T'}\right)_{2,h}} \quad (D14)$$

$$\left(\frac{V_x}{a'_{cr}}\right)_2 = \left(\frac{V_x}{a}\right)_{2,h} \sqrt{\frac{k+1}{2}} \sqrt{\left(\frac{T}{T'}\right)_{2,h}} \quad (D15)$$

$$\frac{V_{u,m}}{a'_{cr}} = \bar{r} \frac{V_{u,h}}{a'_{cr}} \quad (D16)$$

and

$$\frac{U_m}{a'_{cr}} = \frac{1}{\bar{r}} \frac{U_h}{a'_{cr}} \quad (D17)$$

The ratio $(T/T')_{1,m}$ is calculated by equation (D2) written at the mean radius, and the ratio $(T'/T'')_{1,m}$ by

$$\left(\frac{T'}{T''}\right)_{1,m} = \left[1 - \frac{k-1}{k+1} \frac{U}{a'_{cr}} \left(2 \frac{V_u}{a'_{cr}} - \frac{U}{a'_{cr}}\right)\right]_{1,m}^{-1} \quad (D18)$$

Likewise, the ratios

$$\left(\frac{T}{T'}\right)_{2,m} = 1 - \frac{k-1}{k+1} \left[\left(\frac{V_u}{a'_{cr}}\right)_{2,m}^2 + \left(\frac{V_x}{a'_{cr}}\right)_2^2\right] \quad (D19)$$

and

$$\left(\frac{T'}{T''}\right)_{2,m} = \left[1 - \frac{k-1}{k+1} \frac{U}{a'_{cr}} \left(2 \frac{V_u}{a'_{cr}} - \frac{U}{a'_{cr}}\right)\right]_{2,m}^{-1} \quad (D20)$$

Stage-work parameter is calculated by

$$\frac{-gJ\Delta h'}{U_t^2} = \frac{\left[\left(\frac{V_u}{a'_{cr}}\right)_{1,h} - \left(\frac{V_u}{a'_{cr}}\right)_{2,h} \sqrt{\frac{T'_2}{T'_1}}\right] \left(\frac{r_h}{r_t}\right)^2}{\left(\frac{U}{a'_{cr}}\right)_{1,h}} \quad (D21)$$

Exit weight-flow parameter is defined, as in reference 1, as

$$\hat{w}_2 = \frac{W}{A_F(\rho' a'_{cr})_2} \quad (D22)$$

For calculation purposes, equation (D22) is expanded to

$$\hat{w}_2 = \left\{1 - \frac{k-1}{k+1} \left[\left(\frac{V_u}{a'_{cr}}\right)_{2,m}^2 + \left(\frac{V_x}{a'_{cr}}\right)_2^2\right]\right\}^{\frac{1}{k-1}} \left(\frac{V_x}{a'_{cr}}\right)_2 \left[1 - \left(\frac{r_h}{r_t}\right)^2\right] \quad (D23)$$

Turning angle at the hub radius is

$$\Delta\beta_h = \tan^{-1} \left(\frac{\frac{V_x}{a'_{cr}}}{\frac{V_u}{a'_{cr}} - \frac{U}{a'_{cr}}} \right)_{2,h} - \tan^{-1} \left(\frac{\frac{V_x}{a'_{cr}}}{\frac{V_u}{a'_{cr}} - \frac{U}{a'_{cr}}} \right)_{1,h} \quad (D24)$$

REFERENCES

1. Cavicchi, Richard H., and English, Robert E.: A Rapid Method for Use in Design of Turbines within Specified Aerodynamic Limits. NACA TN 2905, 1953.
2. Cavicchi, Richard H., and English, Robert E.: Analysis of Limitations Imposed on One-Spool Turbojet-Engine Designs by Compressors and Turbines at Flight Mach Numbers of 0, 2.0, and 2.8. NACA RM E54F21a, 1954.
3. Ashwood, P. F., and Fletcher, P. J.: The Performance of Some Typical Turbojet Engine Exhaust Systems, with Particular Reference to the Effects of Swirl. Rep. No. R.112, British N.G.T.E., Dec. 1951.
4. Lieblein, Seymour, Schwenk, Francis C., and Broderick, Robert L.: Diffusion Factor for Estimating Losses and Limiting Blade Loadings in Axial-Flow-Compressor Blade Elements. NACA RM E53D01, 1953.
5. Alpert, Sumner, and Litrenta, Rose M.: Construction and Use of Charts in Design Studies of Gas Turbines. NACA TN 2402, 1951.
6. English, Robert E., and Cavicchi, Richard H.: Possible Range of Design of One-Spool Turbojet Engines Within Specified Turbine-Design Limits. Preprint No. 53-S-33, A.S.M.E., 1953.
7. LaValle, Vincent L., and Huppert, Merle C.: Effects of Several Design Variables on Turbine-Wheel Weight. NACA TN 1814, 1949.

TABLE I. - SUMMARY OF DESIGN PARAMETERS USED IN CHARTS

Rows of downstream stator blades	$D_{2,h}$	$D_{3,h}$	$\bar{\omega}$	Chart
1	0.2	----	0.05	I(a)1, (b)1
	.4	----	.05	I(a)2, (b)2
	.6	----	.07	I(a)3, (b)3
	.4	----	.05	II(a)1, (b)1
	.6	----	.07	II(b)1, (b)2
2	0.2	0.2	0.05	III(a)1, (b)1
	.4	.4	.05	III(a)2, (b)2
	.2	.2	.05	IV(a)1, (b)1
	.4	.4	.05	IV(a)2, (b)2
	.6	.6	.07	IV(a)3, (b)3

For charts I and III: $(A_2/A_1)_s = 0.8, 0.9, 1.0, 1.1$

II and IV: $\Delta\beta_h = 120^\circ$

III and IV: $\sigma_{3,m} = 1.5$

For all charts: $M_{1,h} = 0.8$

$(V_x/a)_{2,h} = 0.7$

$r_h/r_t = 0.5, 0.6, 0.7, 0.8, 0.9$

$\sigma_{2,m} = 1.5$

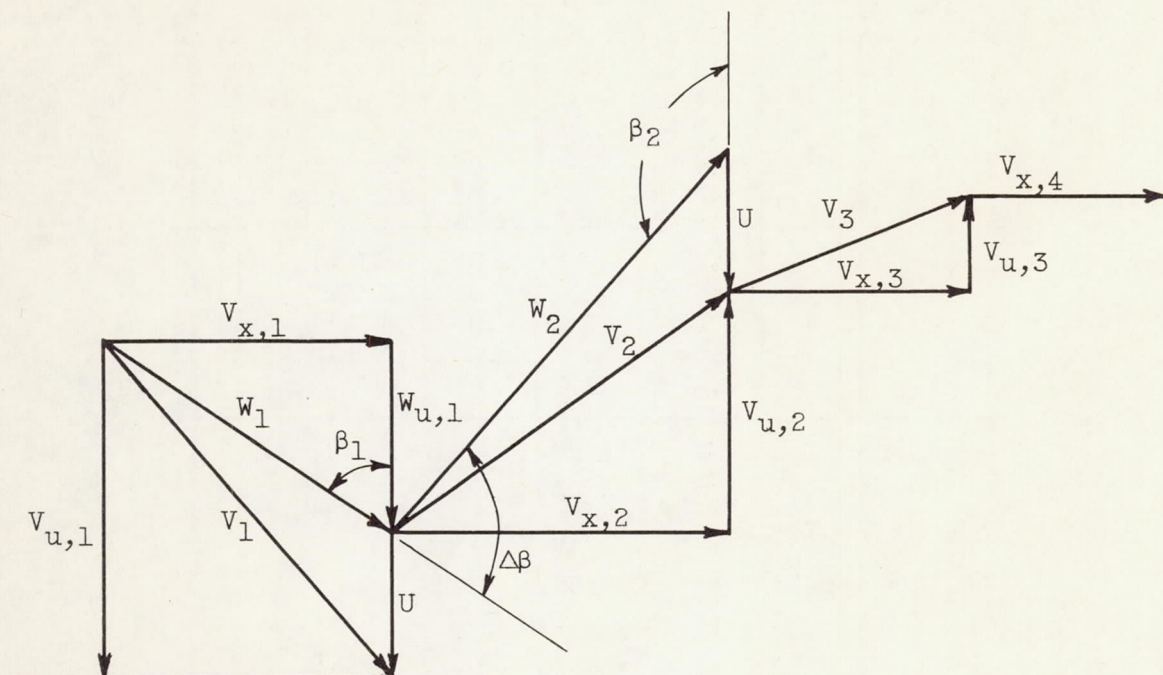


Figure 1. - Velocity diagram of turbine with two rows of downstream stator blades.

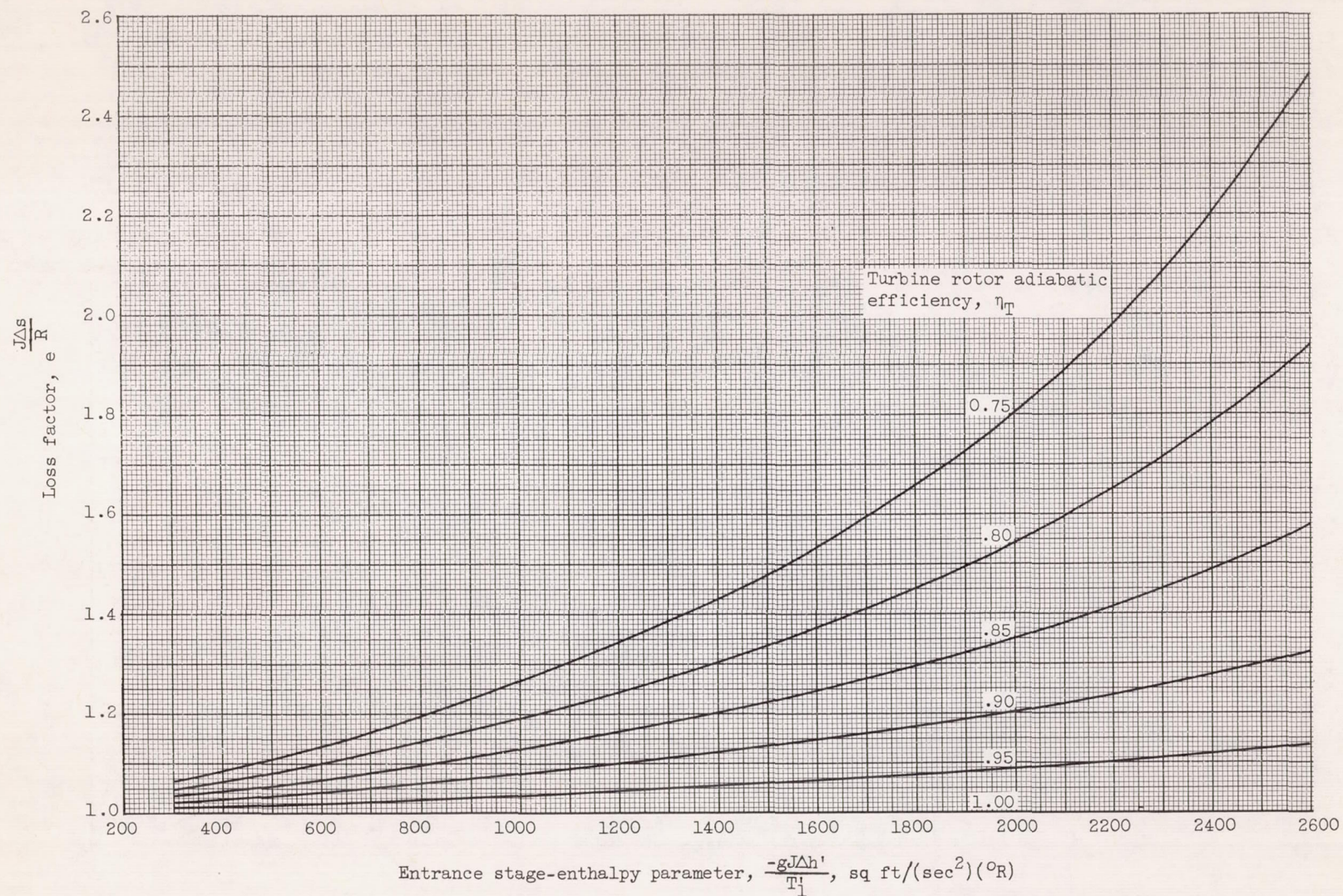
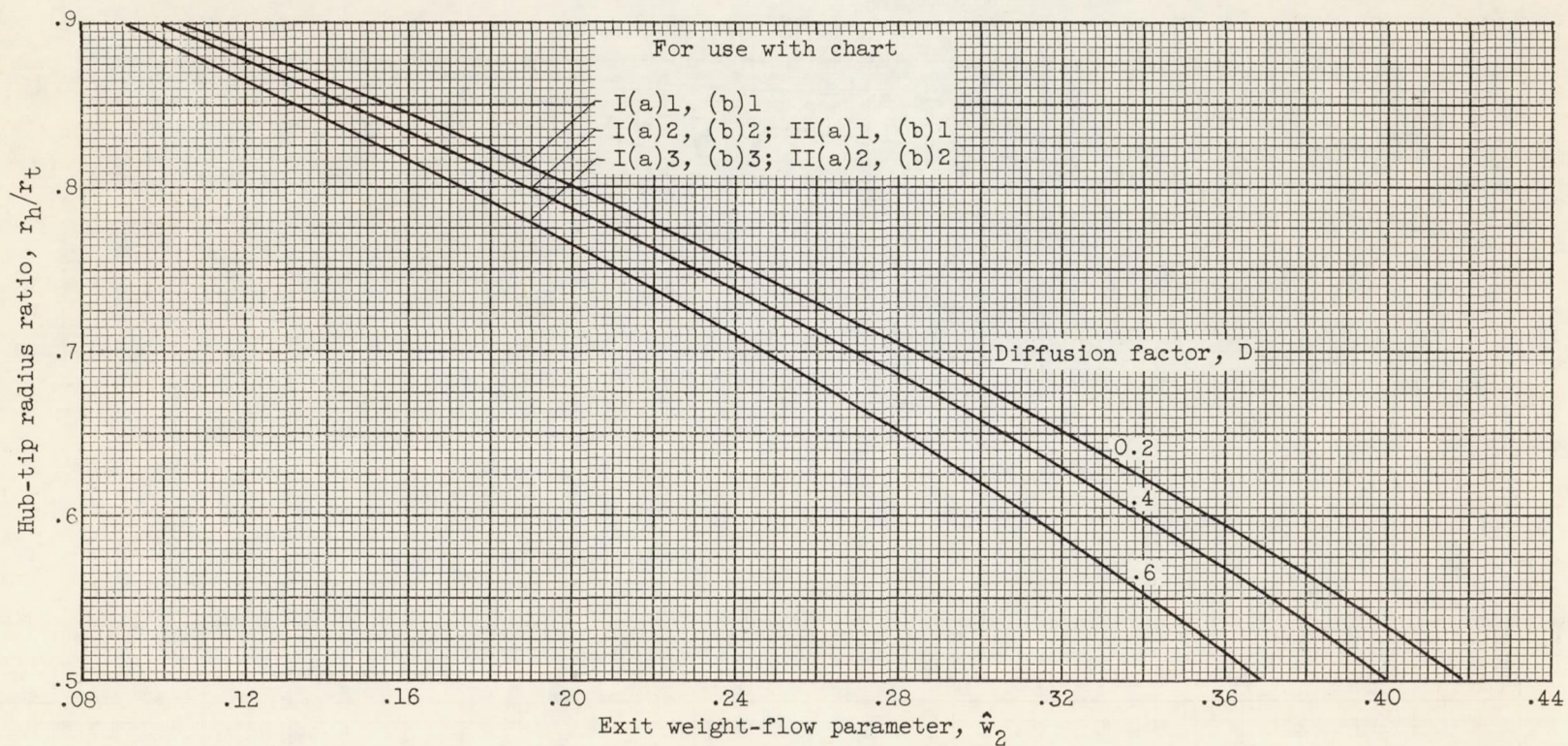
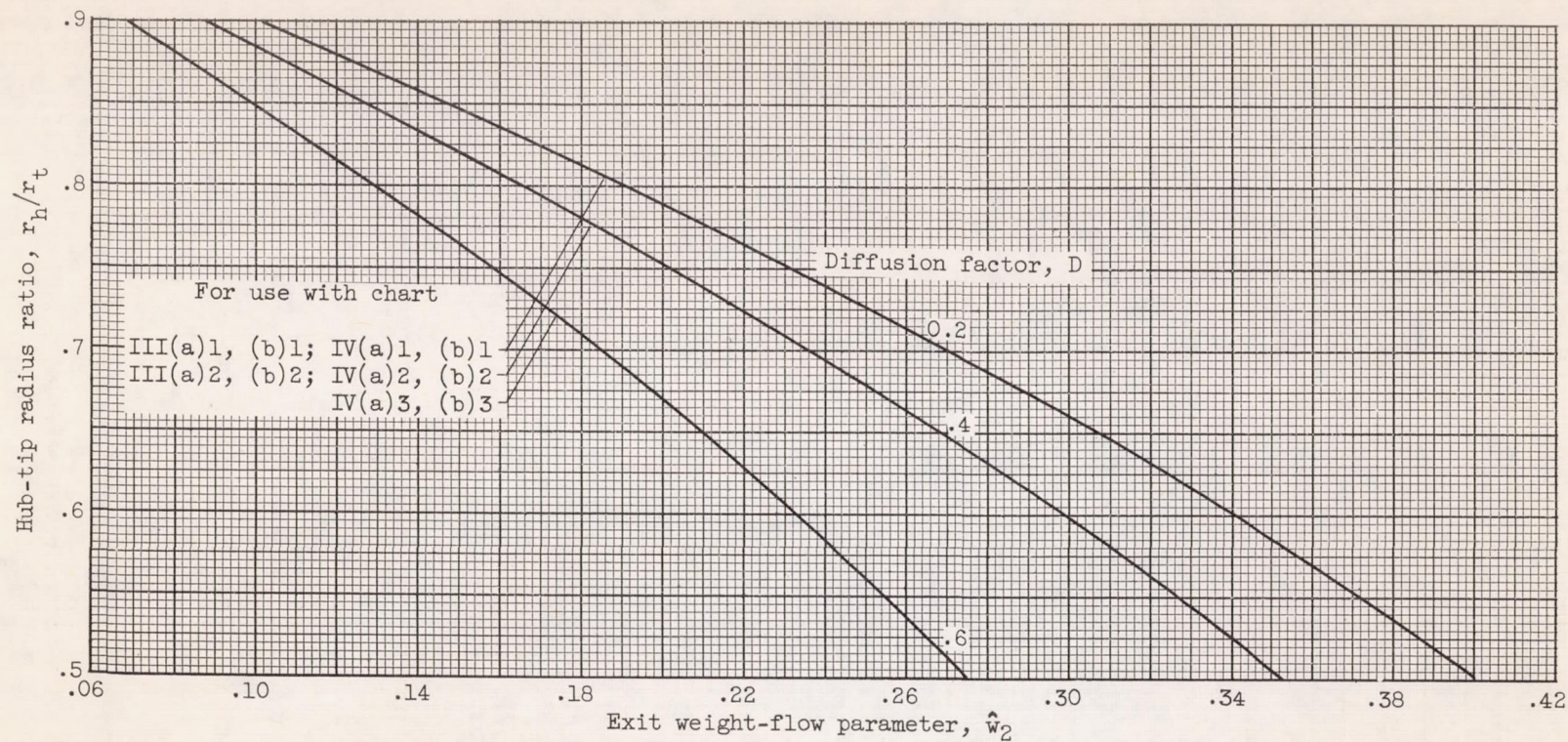


Figure 2. - Variation of loss factor with turbine rotor adiabatic efficiency.



(a) Turbines with one downstream stator.

Figure 3. - Variation of hub-tip radius ratio with exit weight-flow parameter.



(b) Turbines with two downstream stators.

Figure 3. - Concluded. Variation of hub-tip radius ratio with exit weight-flow parameter.

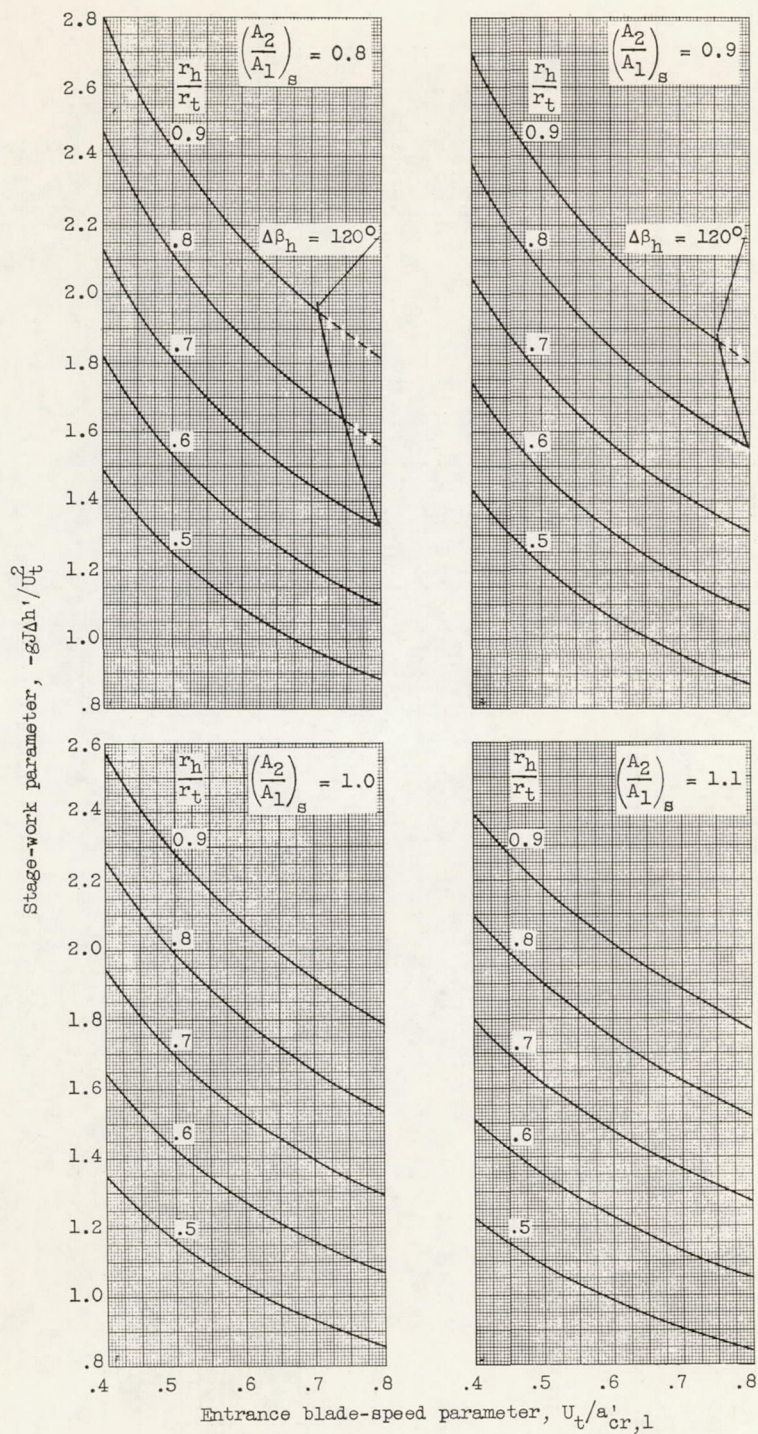
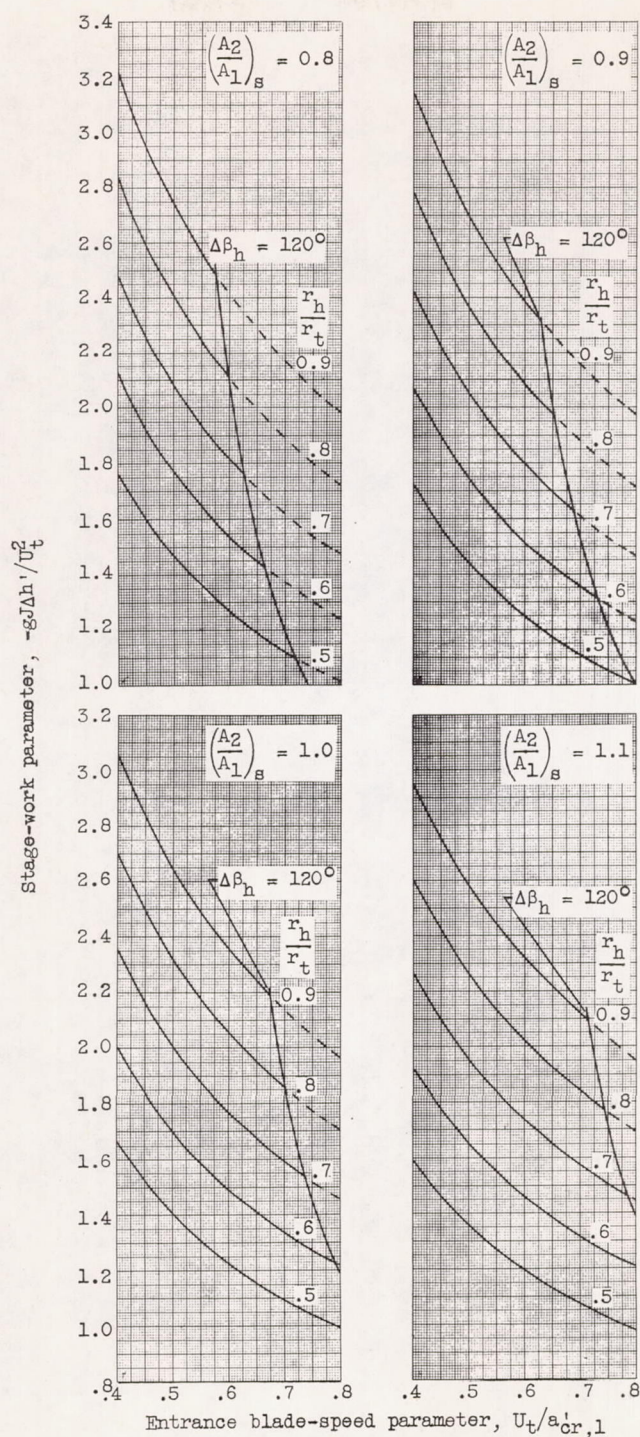
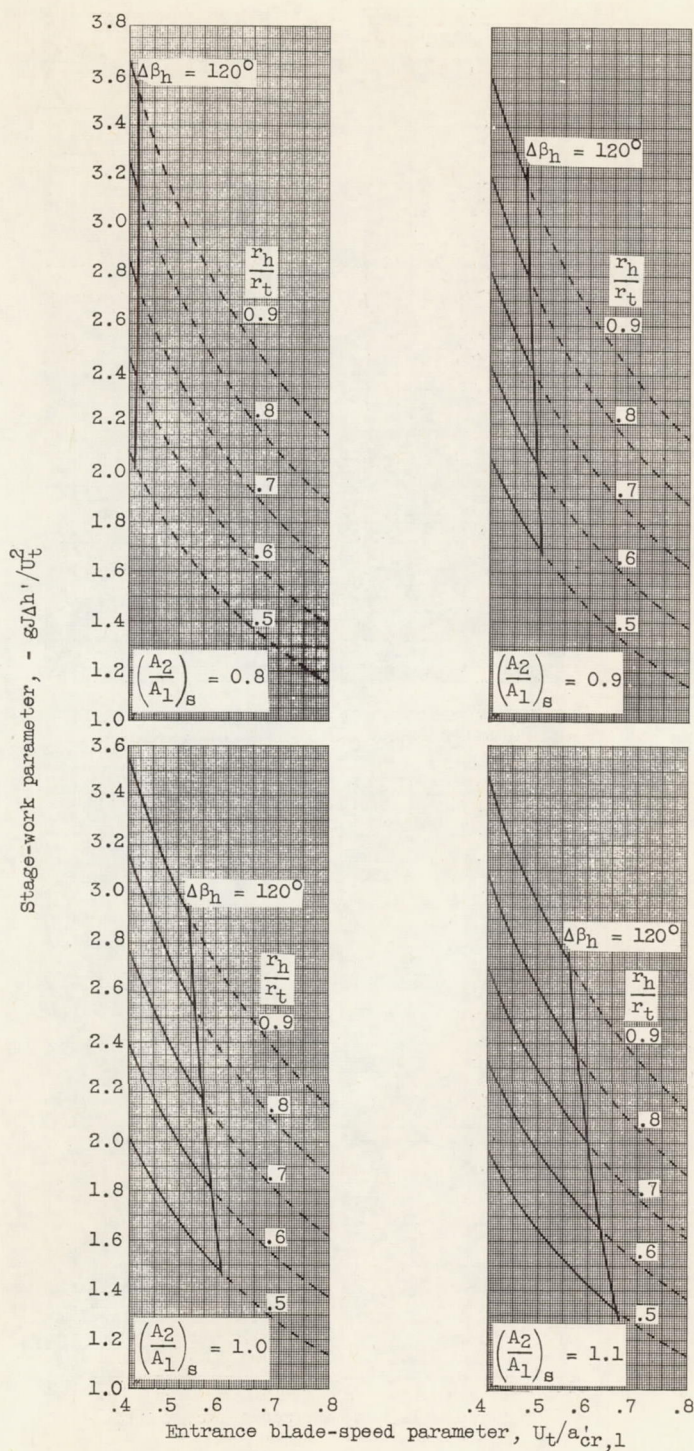


Chart I. - One-stage turbine with one downstream stator.
 (A large working copy of this chart may be obtained by
 using the request card bound in the back of the
 report.)



(a) Continued. Stage-work parameter.

Chart I. - Continued. One-stage turbine with one downstream stator. (A large working copy of this chart may be obtained by using the request card bound in the back of the report.)



3. Diffusion factor, 0.6.

(a) Concluded. Stage-work parameter.

Chart I. - Continued. One-stage turbine with one downstream stator. (A large working copy of this chart may be obtained by using the request card bound in the back of the report.)

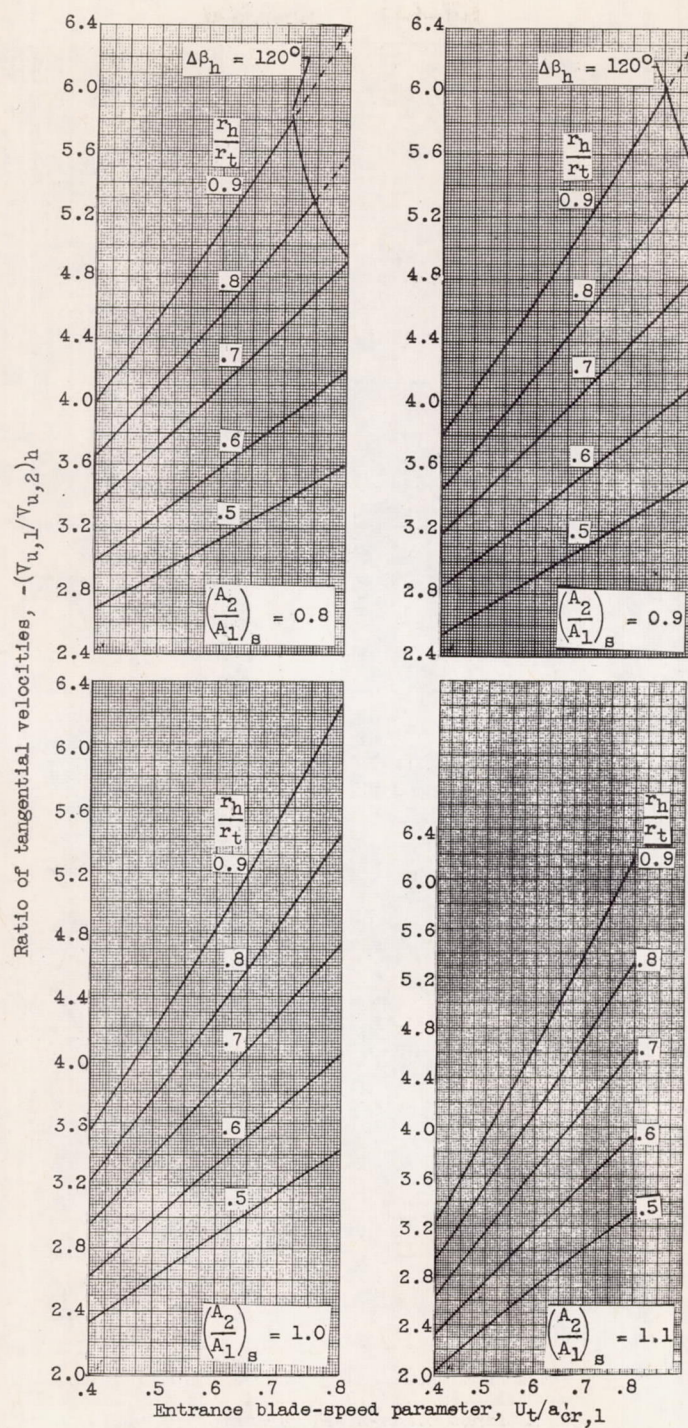
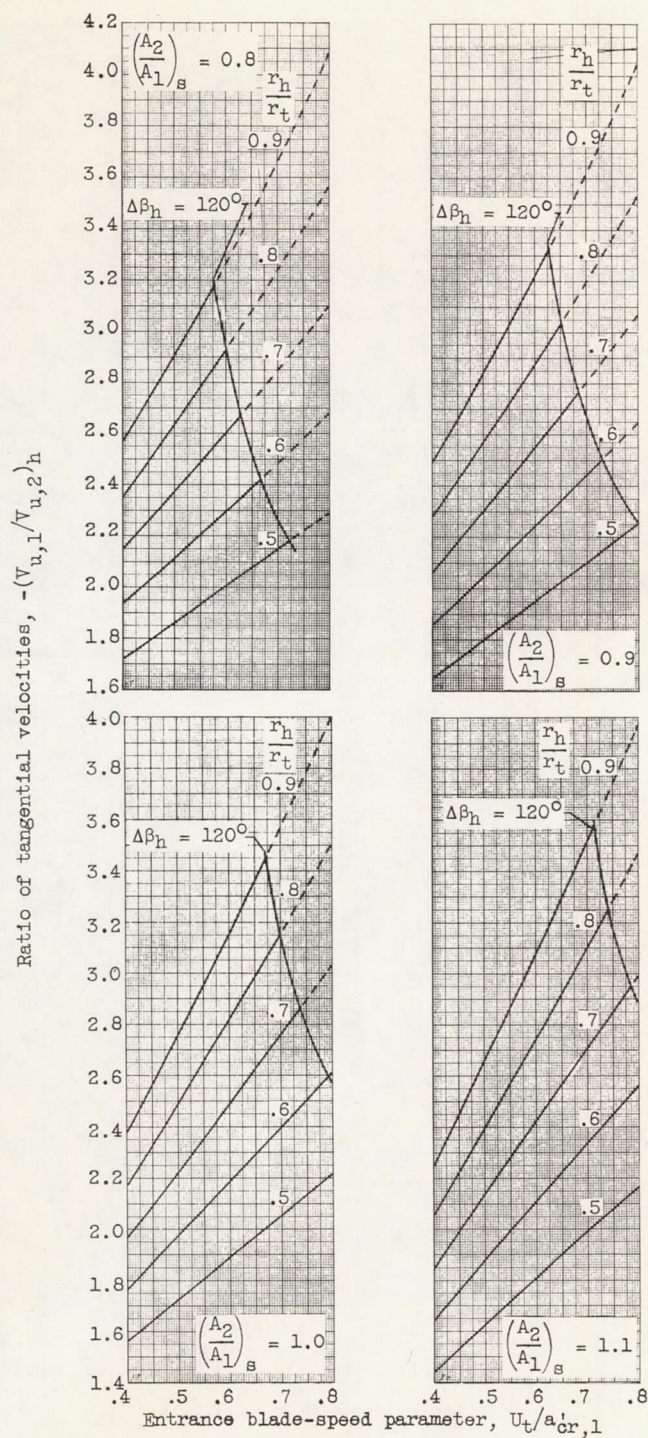
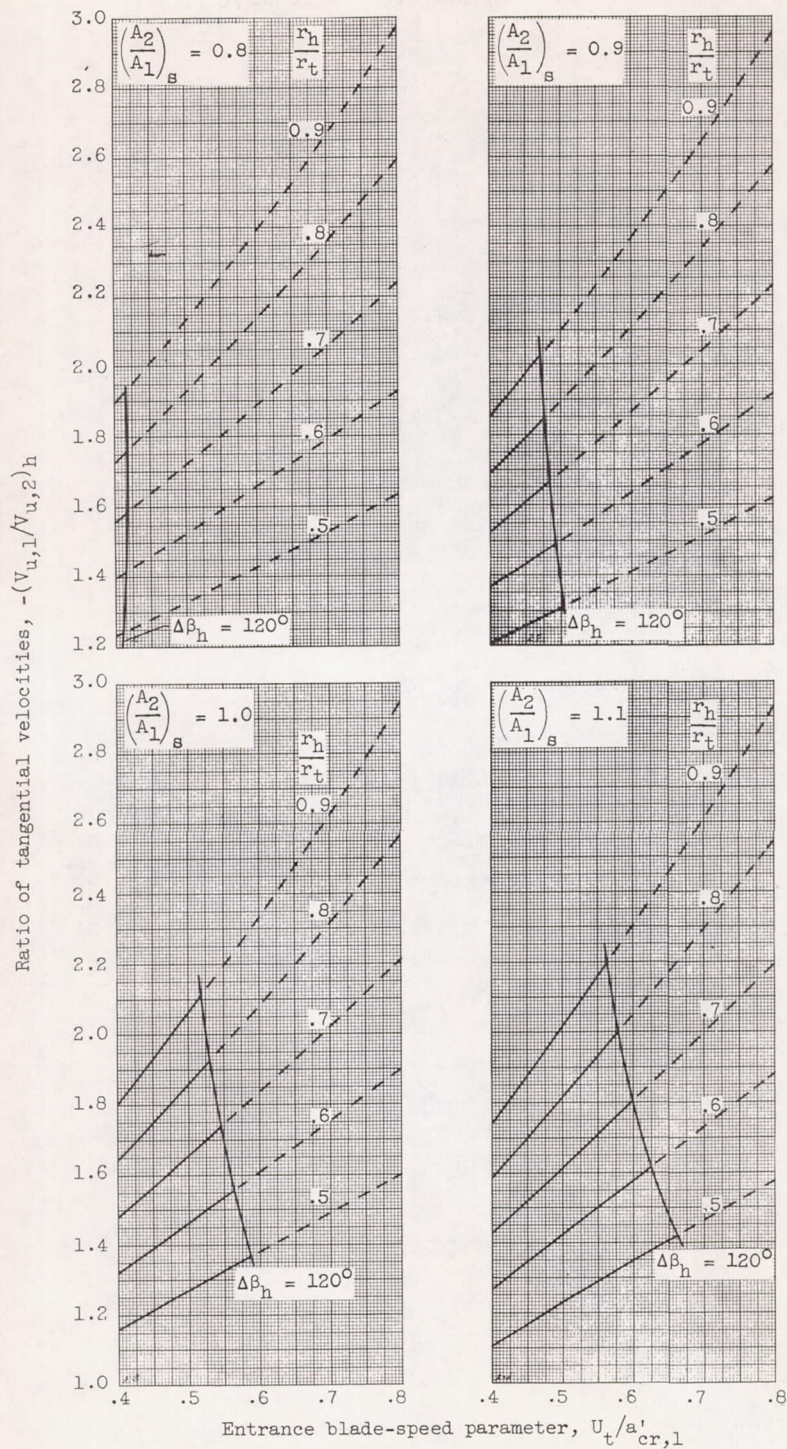


Chart I. - Continued. One-stage turbine with one downstream stator. (A large working copy of this chart may be obtained by using the request card bound in the back of the report.)



(b) Continued. Ratio of tangential velocities.

Chart I. - Continued. One-stage turbine with one downstream stator. (A large working copy of this chart may be obtained by using the request card bound in the back of the report.)



3. Diffusion factor, 0.6 .

(b) Concluded. Ratio of tangential velocities.

Chart I. - Concluded. One-stage turbine with one downstream stator.
(A large working copy of this chart may be obtained by using the request card bound in the back of the report.)

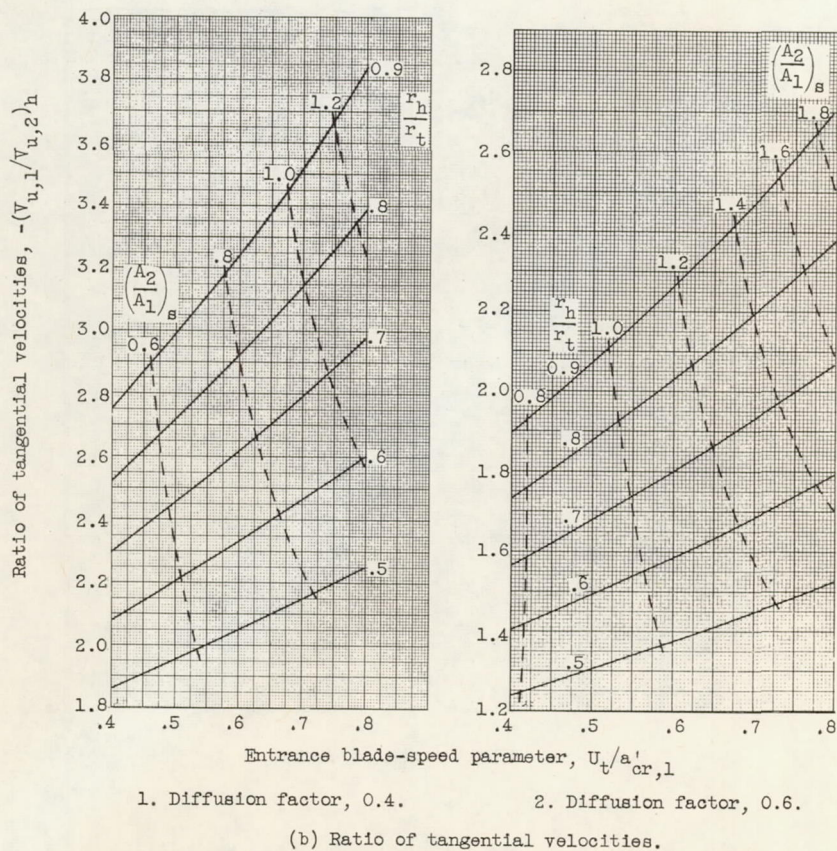
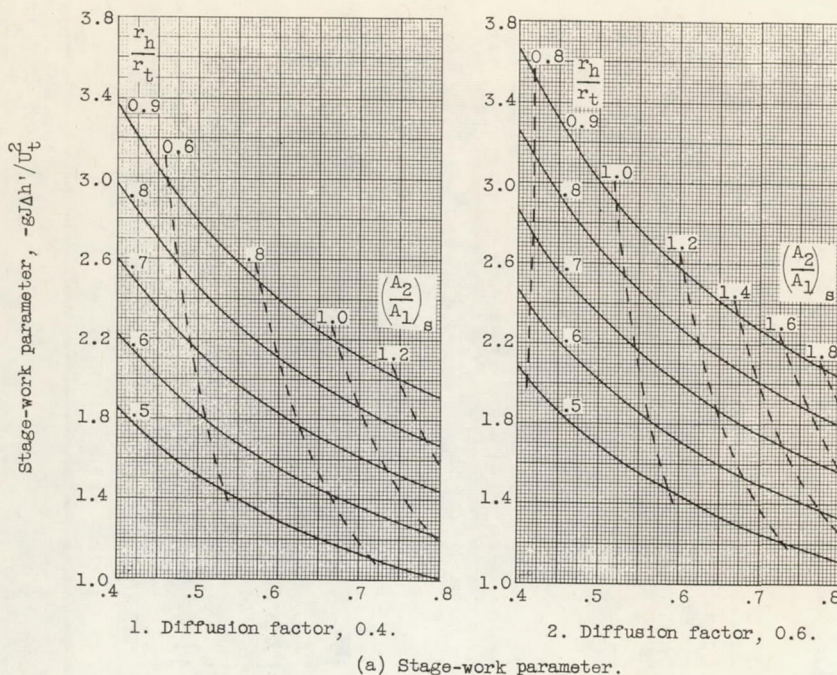
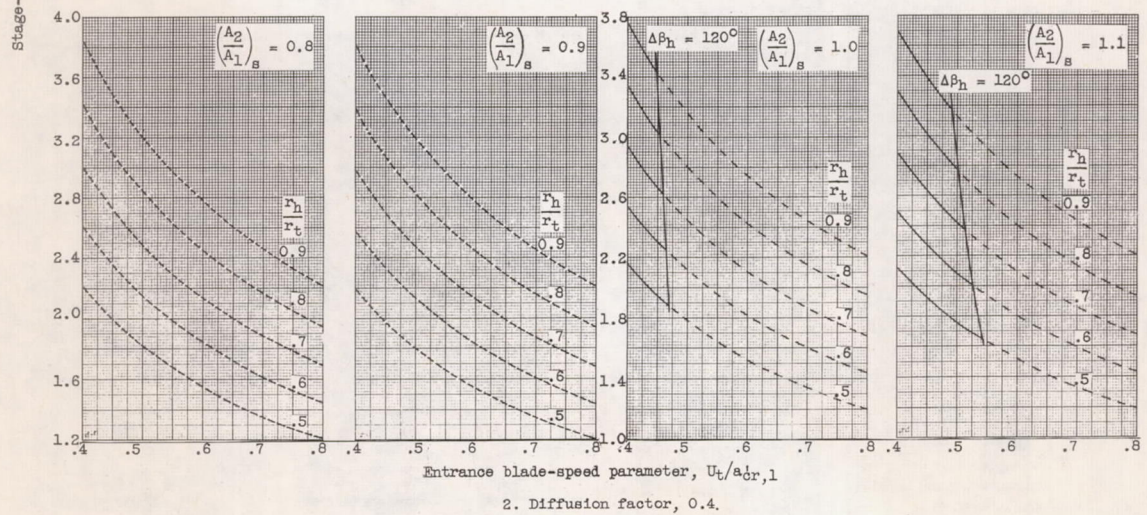
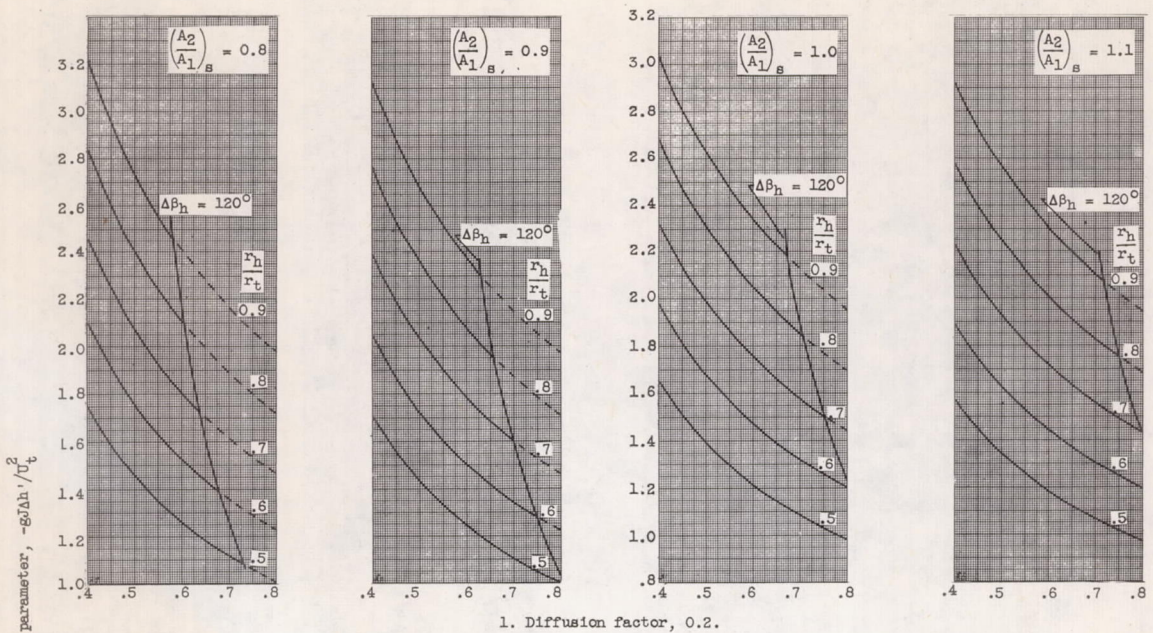


Chart II. - One-stage turbine with one downstream stator and a rotor relative turning angle of 120° at hub radius. (A large working copy of this chart may be obtained by using the request card bound in the back of the report.)



(a) Stage-work parameter.

Chart III. - One-stage turbine with two downstream stators. (A large working copy of this chart may be obtained by using the request card bound in the back of the report.)

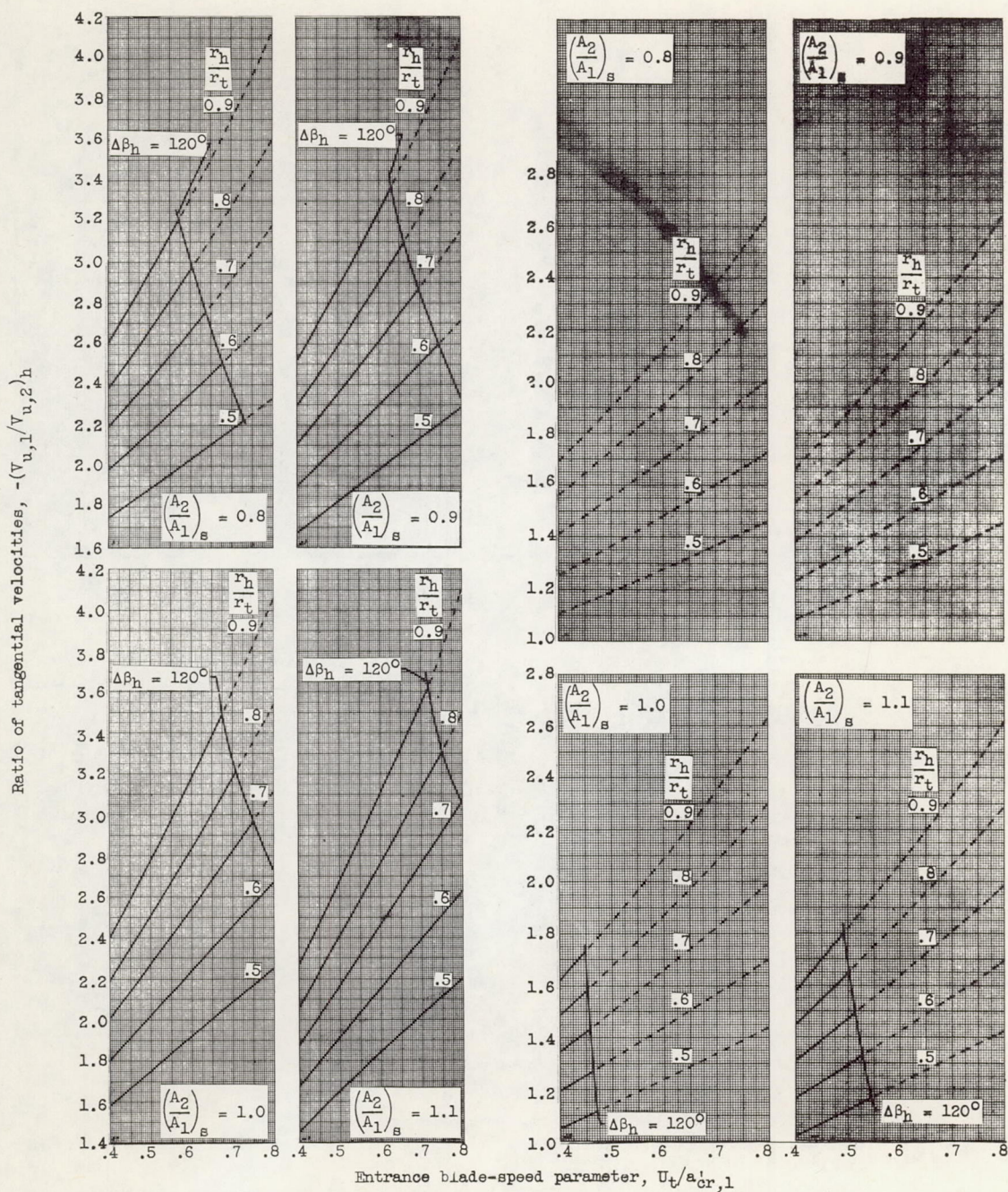


Chart III. - Concluded. One-stage turbine with two downstream stators. (A large working copy of this chart may be obtained by using the request card bound in the back of the report.)

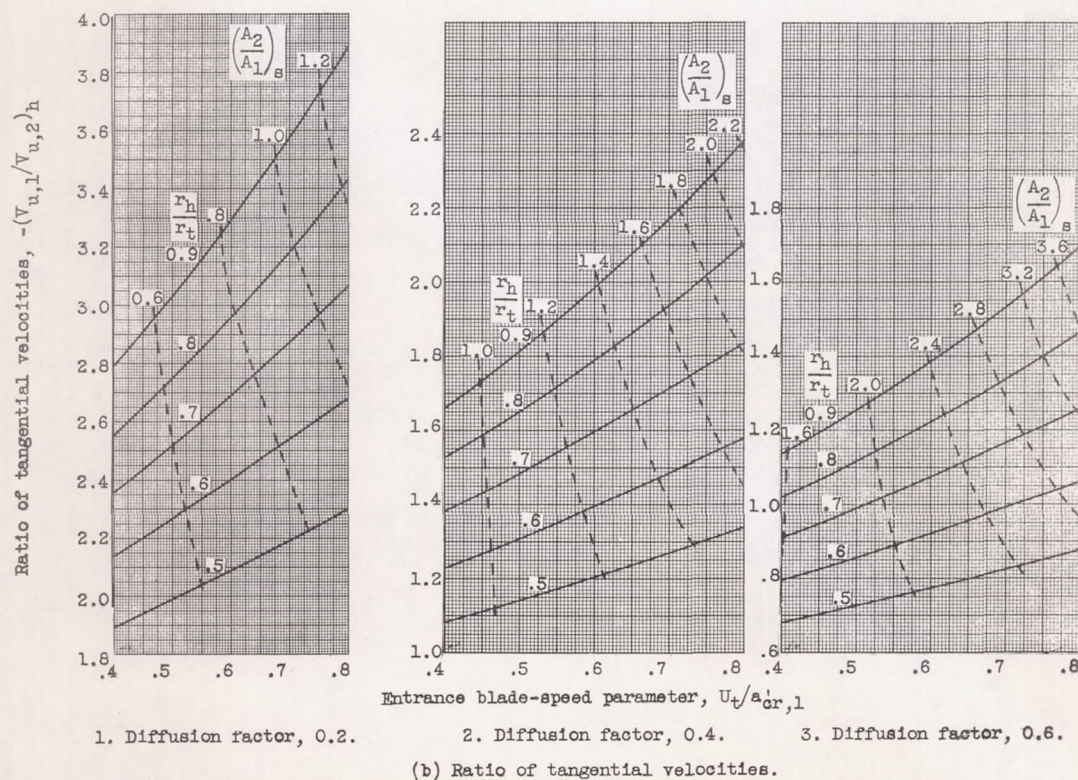
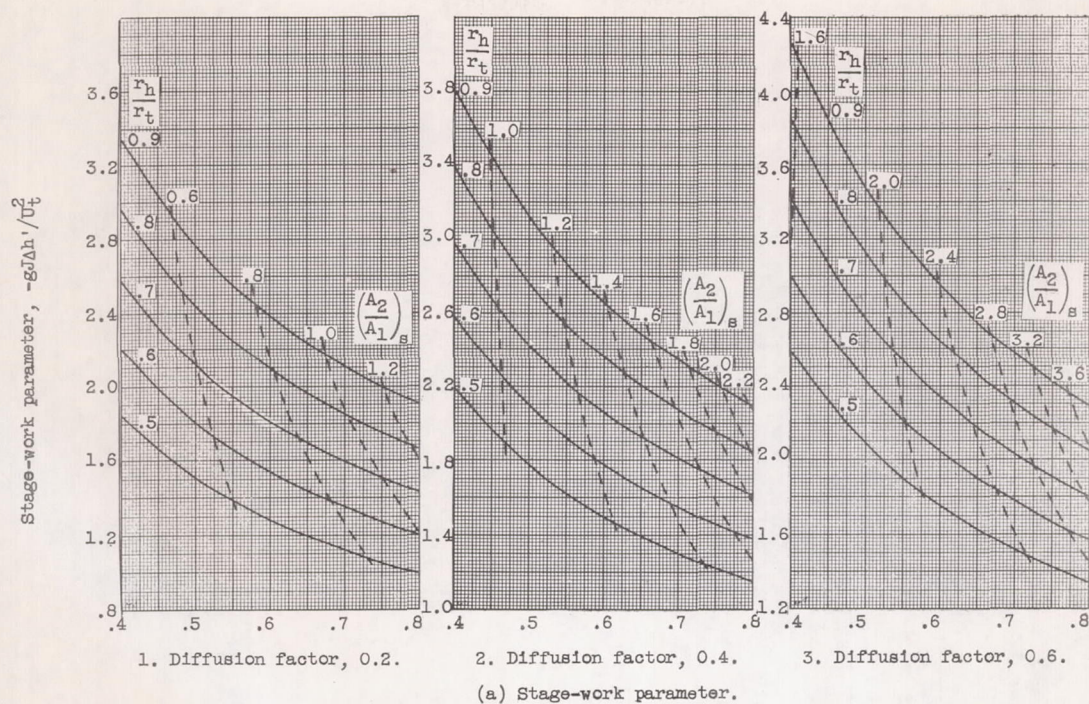
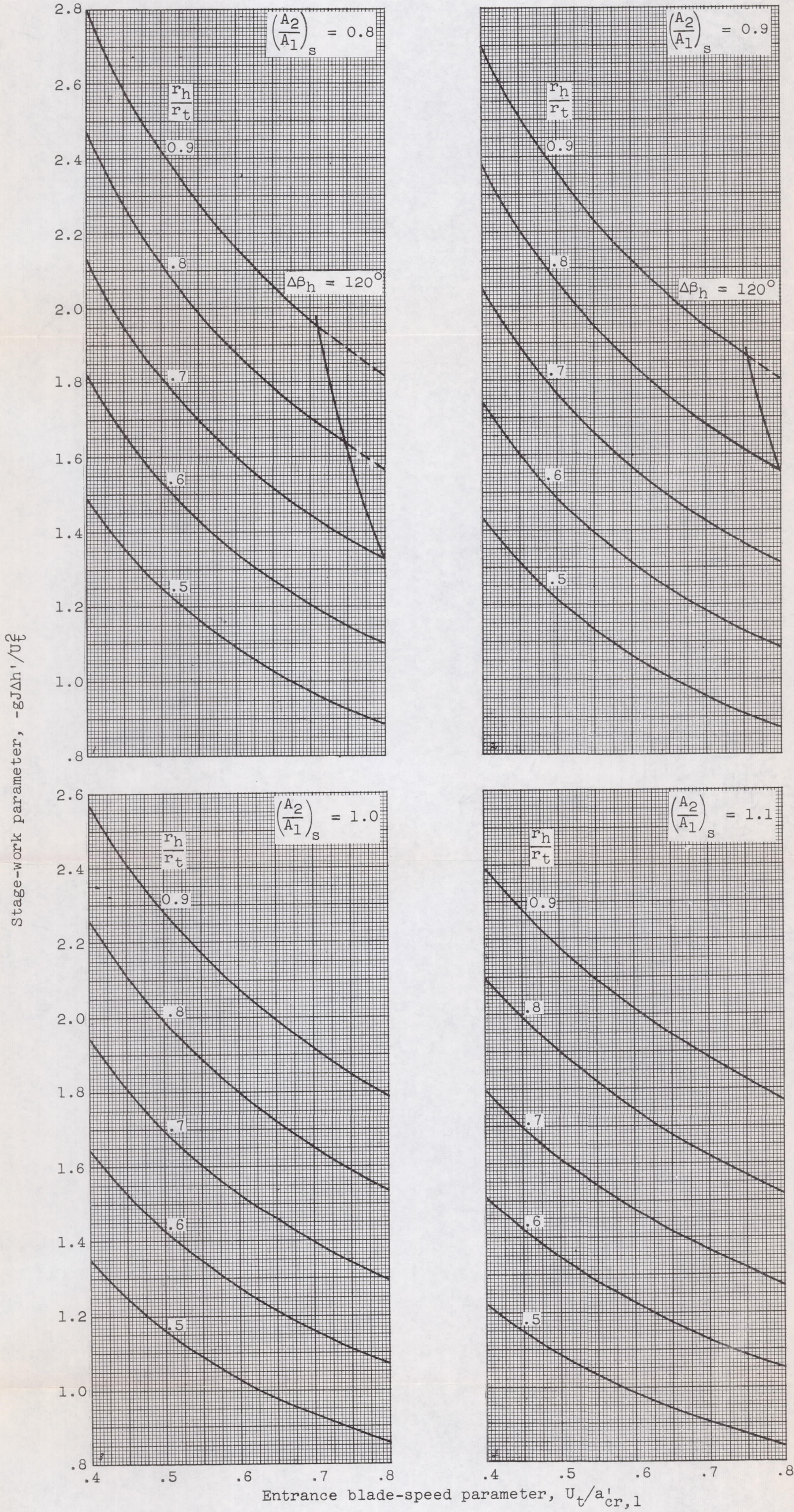
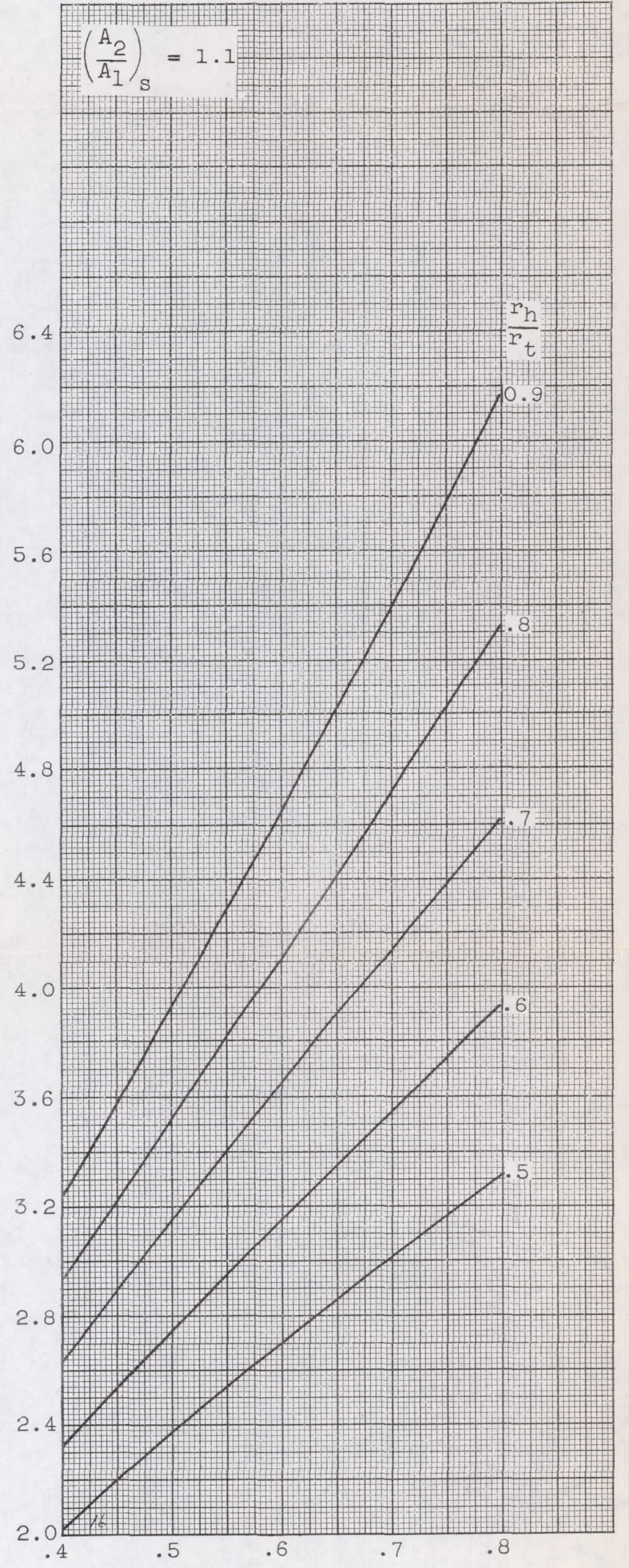
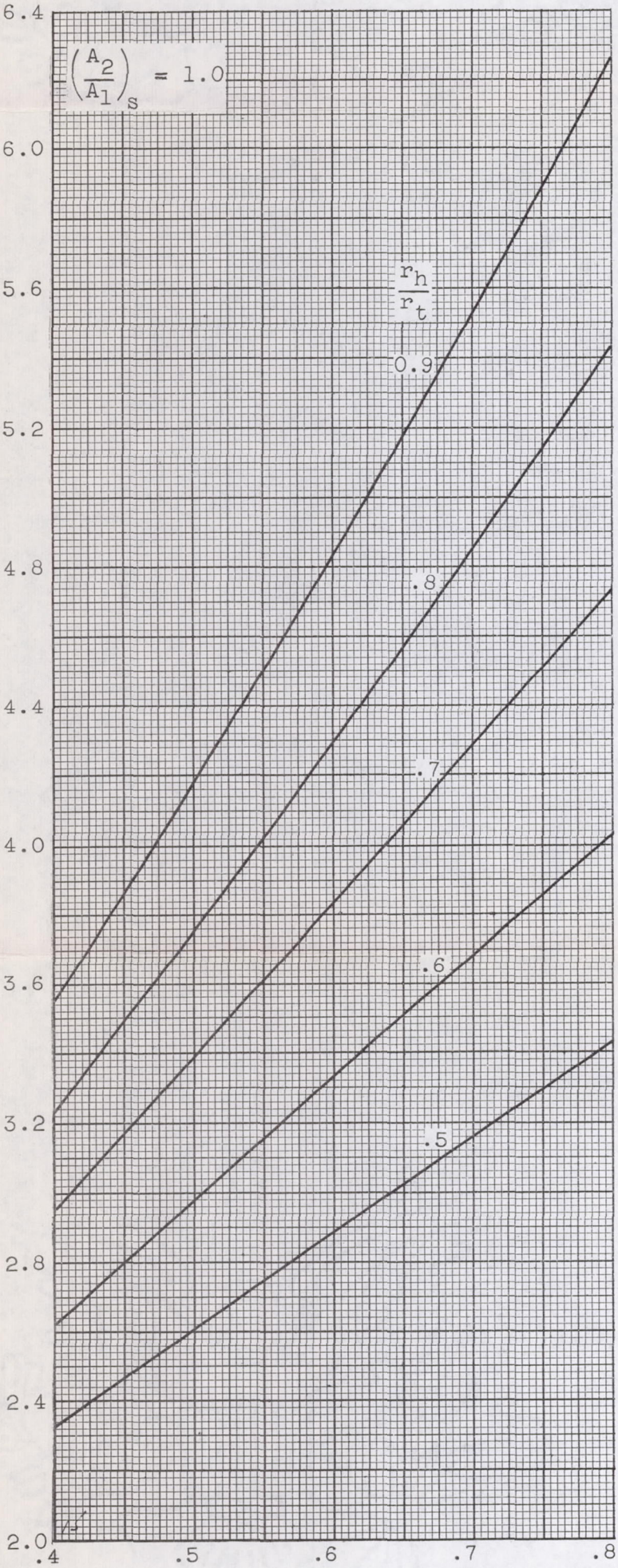
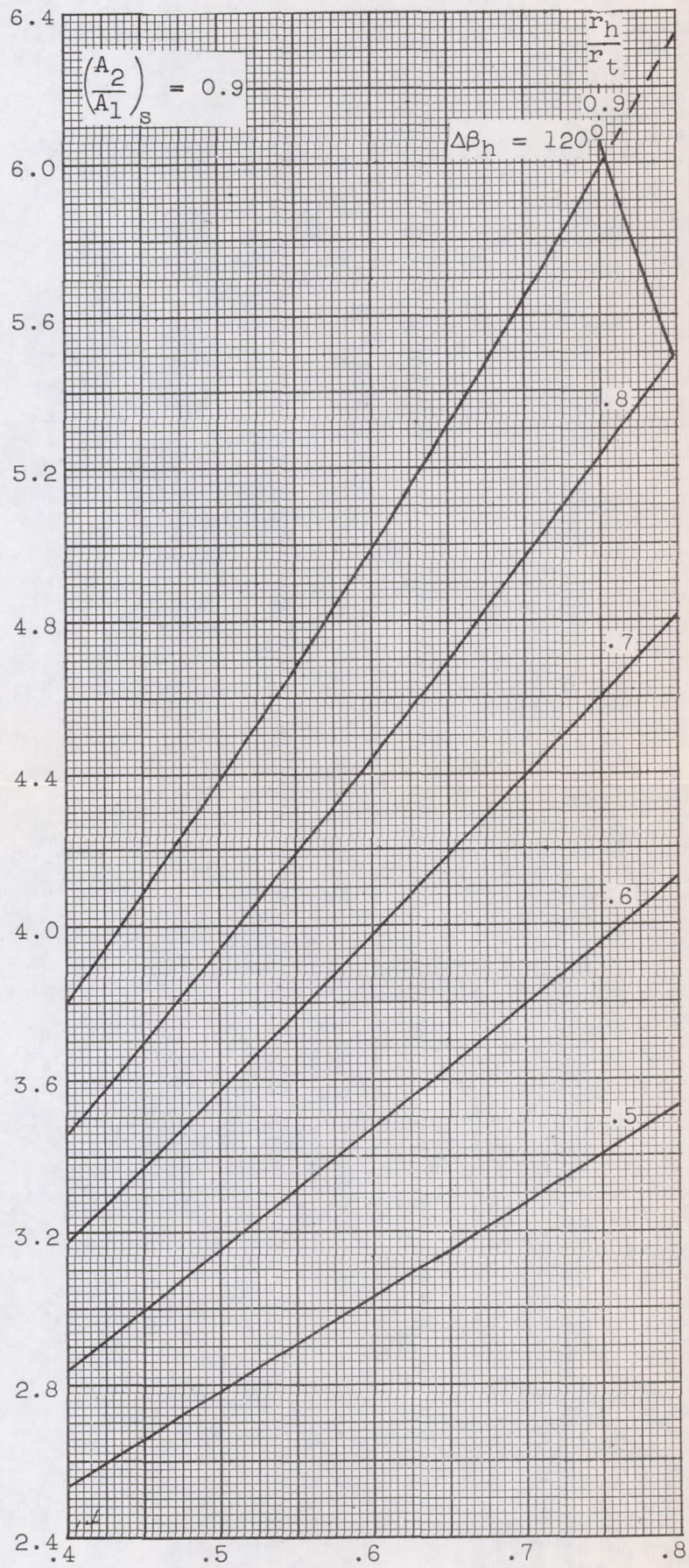
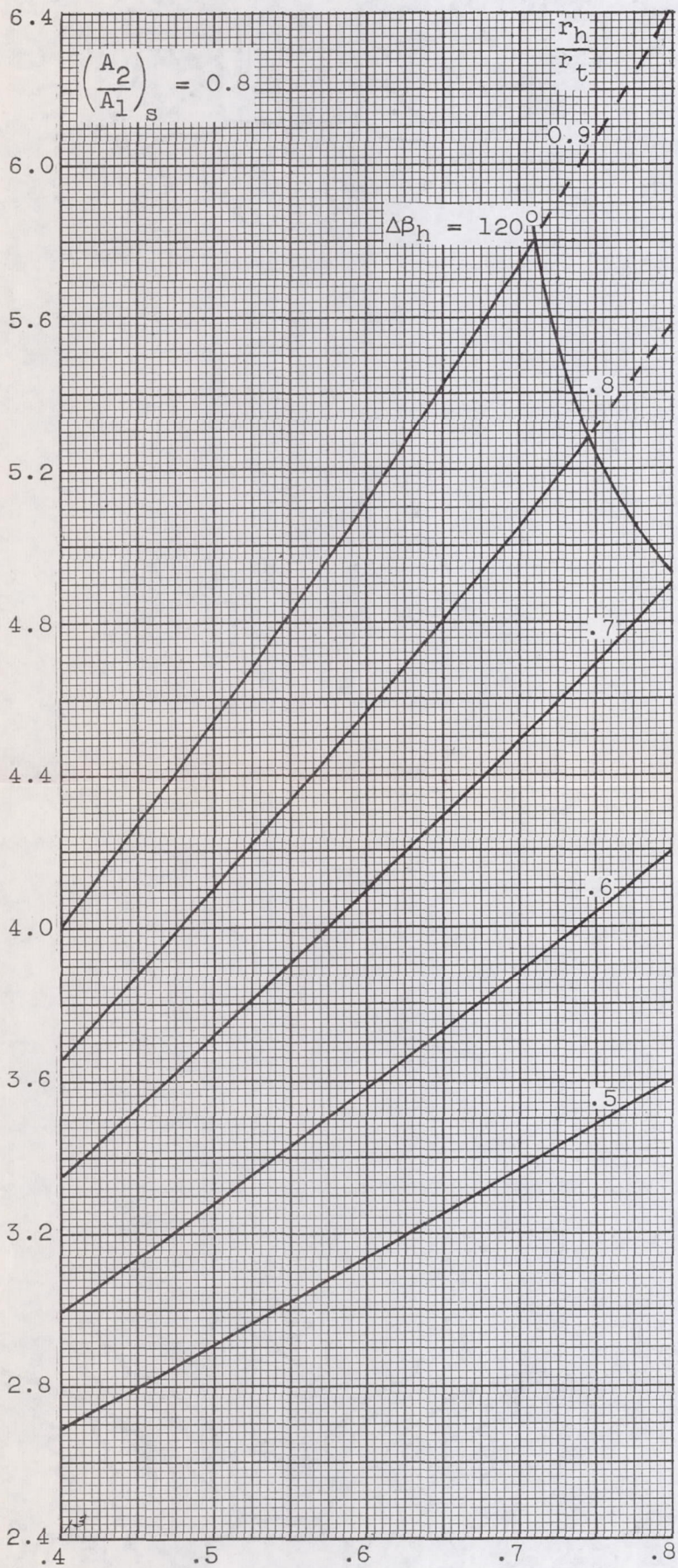


Chart IV. - One-stage turbine with two downstream stators and a rotor relative turning angle of 120° at hub radius. (A large working copy of this chart may be obtained by using the request card bound in the back of the report.)



Ratio of tangential velocities, $-(v_{u,1}/v_{u,2})_h$

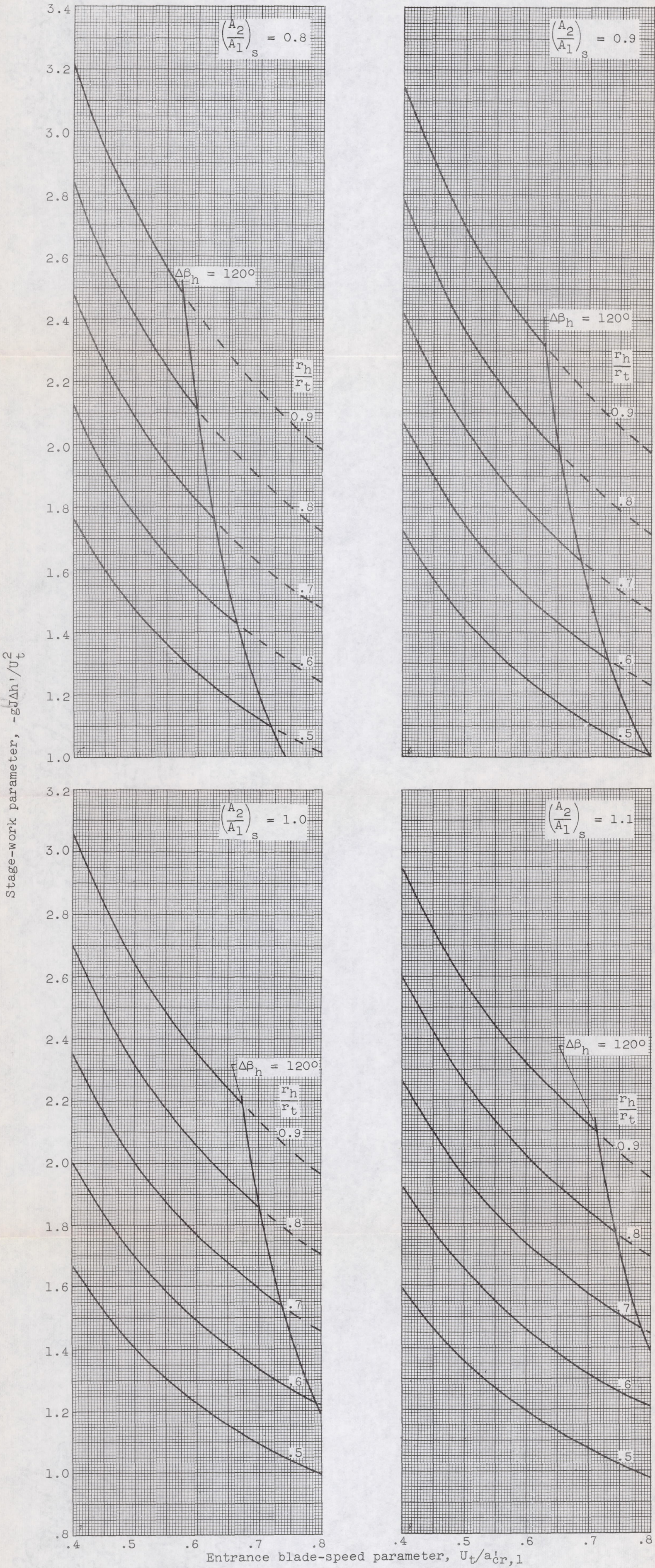


Entrance blade-speed parameter, $U_t/a'_{cr,1}$

1. Diffusion factor, 0.2.

(b) Ratio of tangential velocities.

Chart I. - Continued. One-stage turbine with one downstream stator.

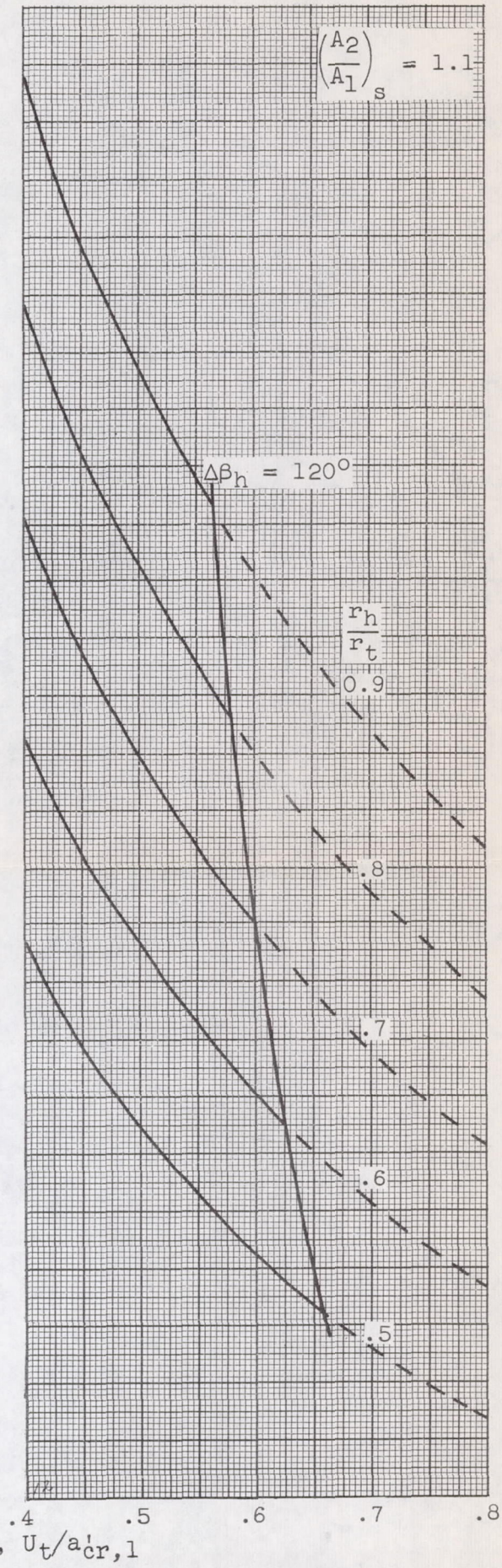
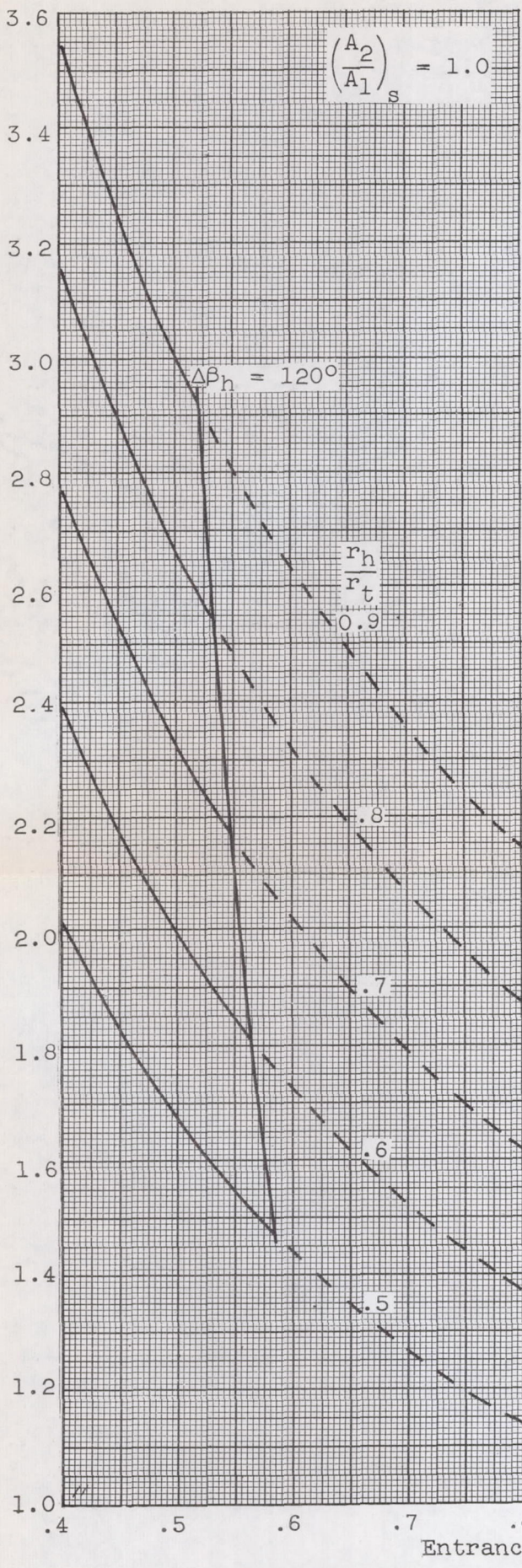
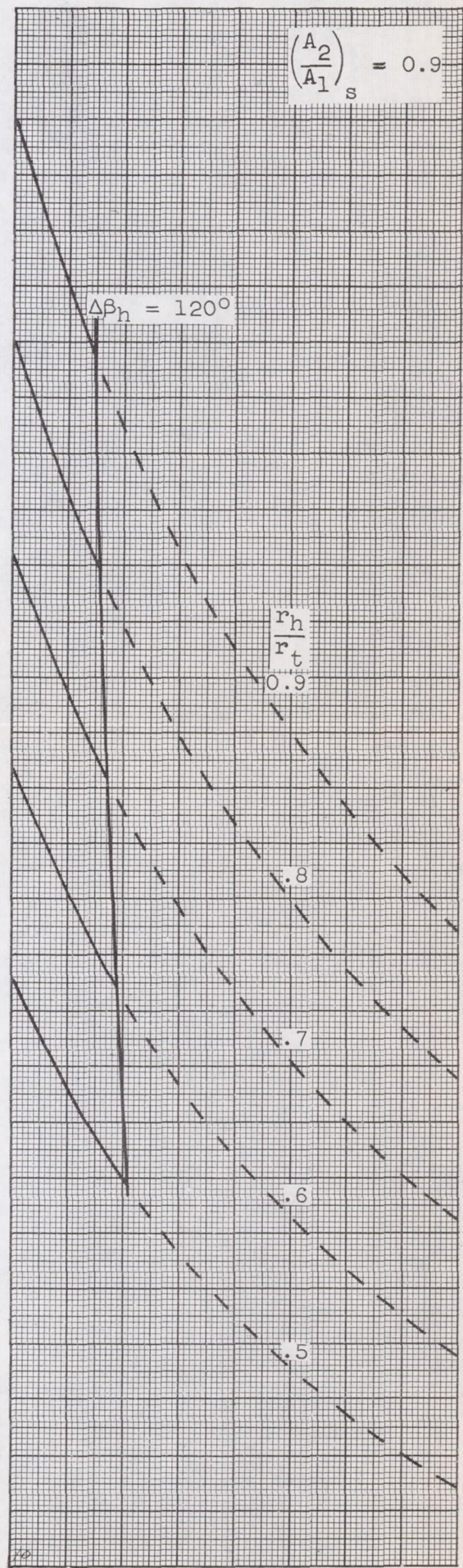
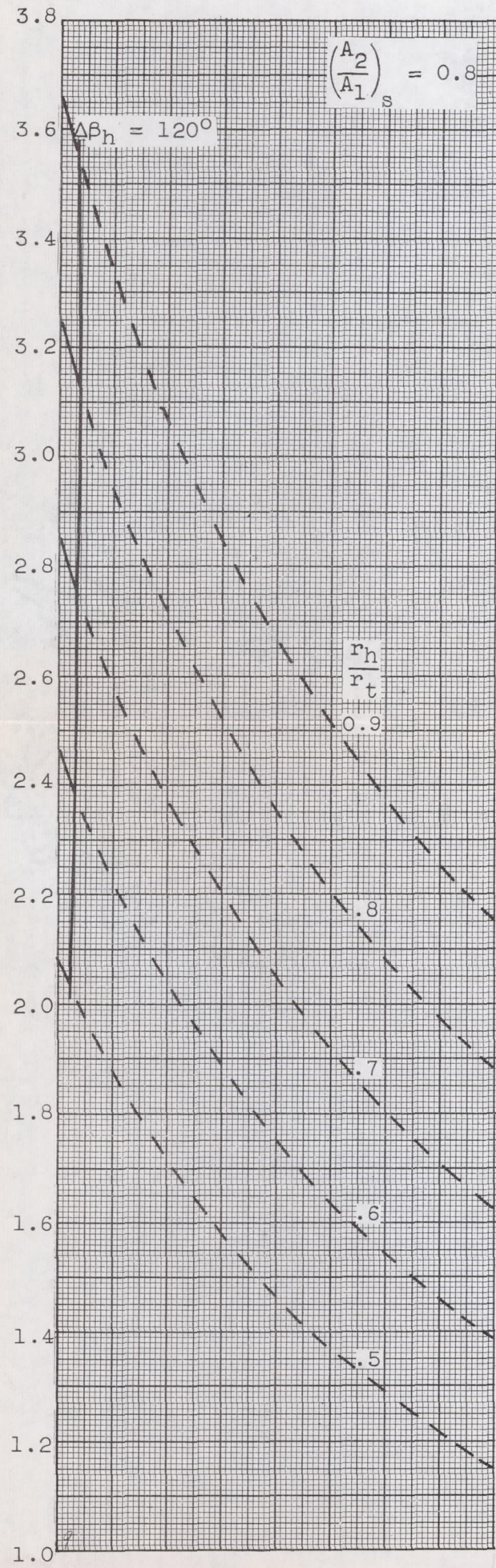


2. Diffusion factor, 0.4.

(a) Continued. Stage-work parameter.

Chart I. - Continued. One-stage turbine with one downstream stator.

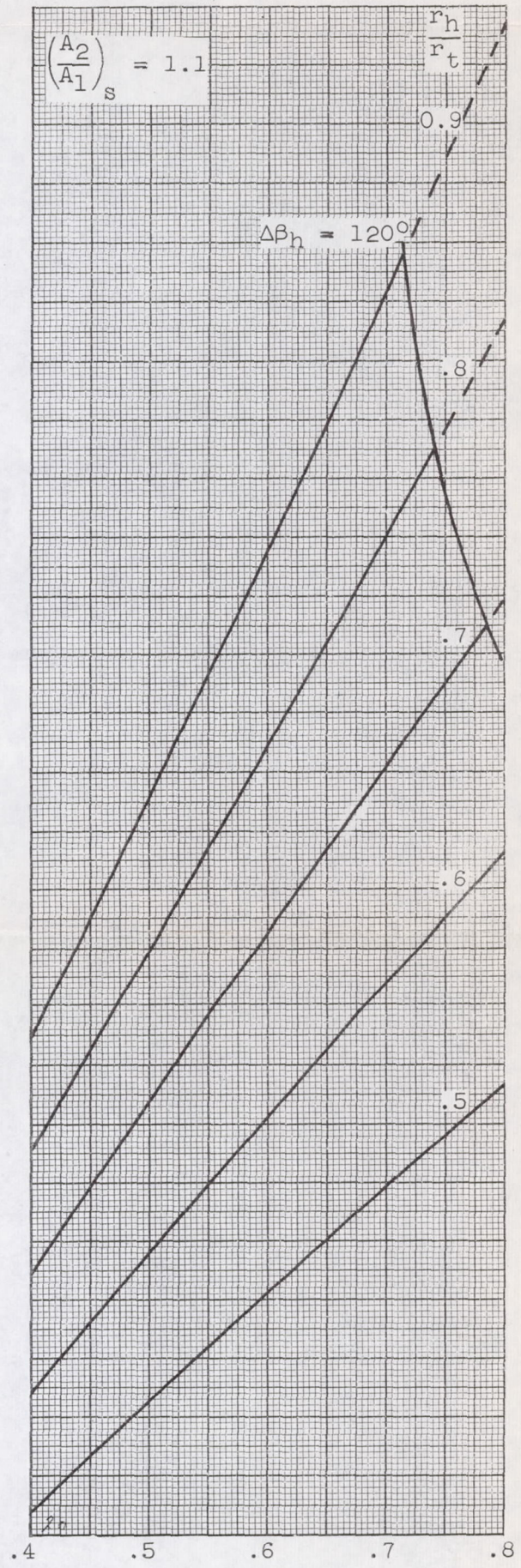
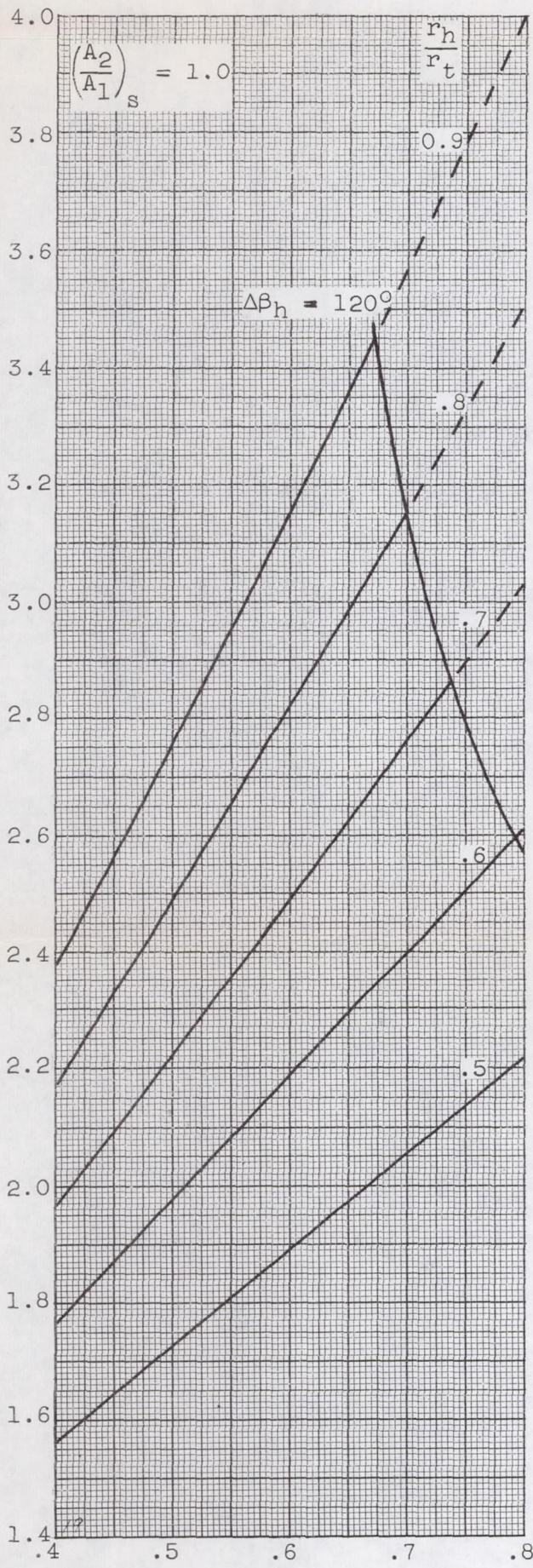
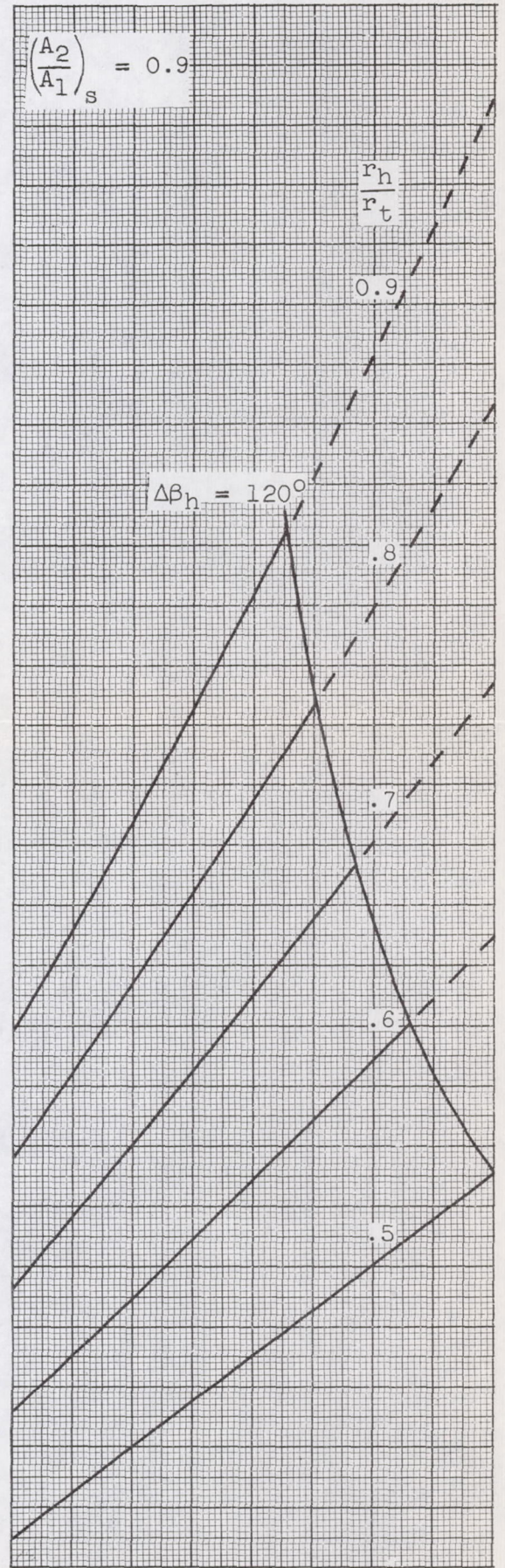
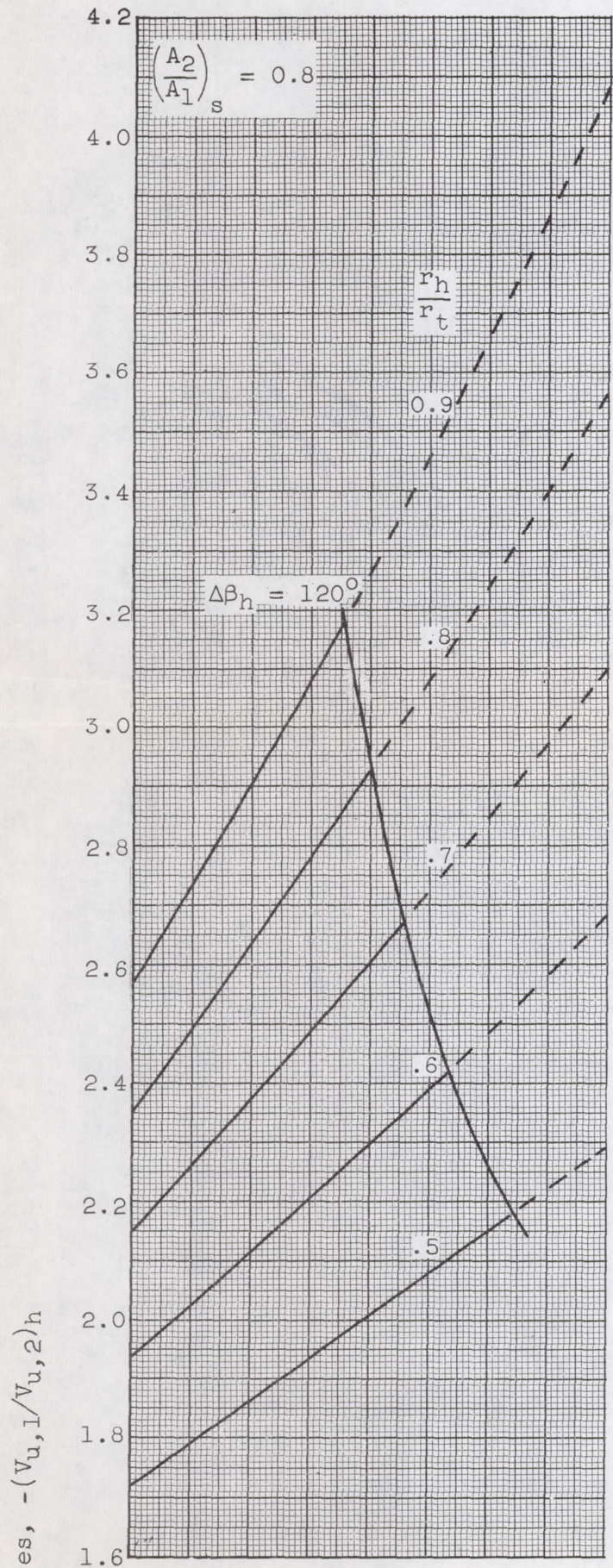
Stage-work parameter, $-gJ\Delta h'/U_t^2$



3. Diffusion factor, 0.6.

(a) Concluded. Stage-work parameter.

Chart I. - Continued. One-stage turbine with one downstream stator.

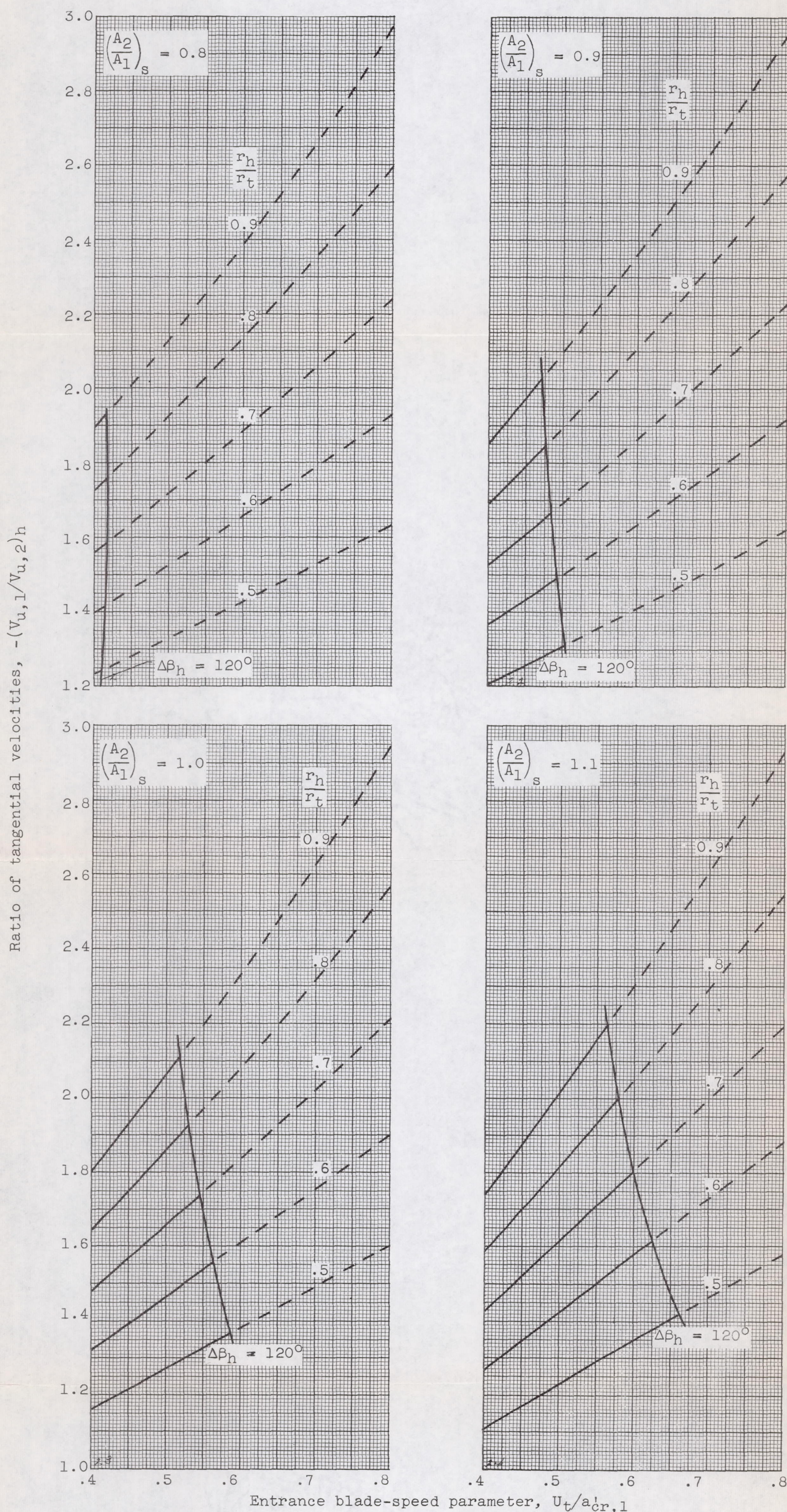


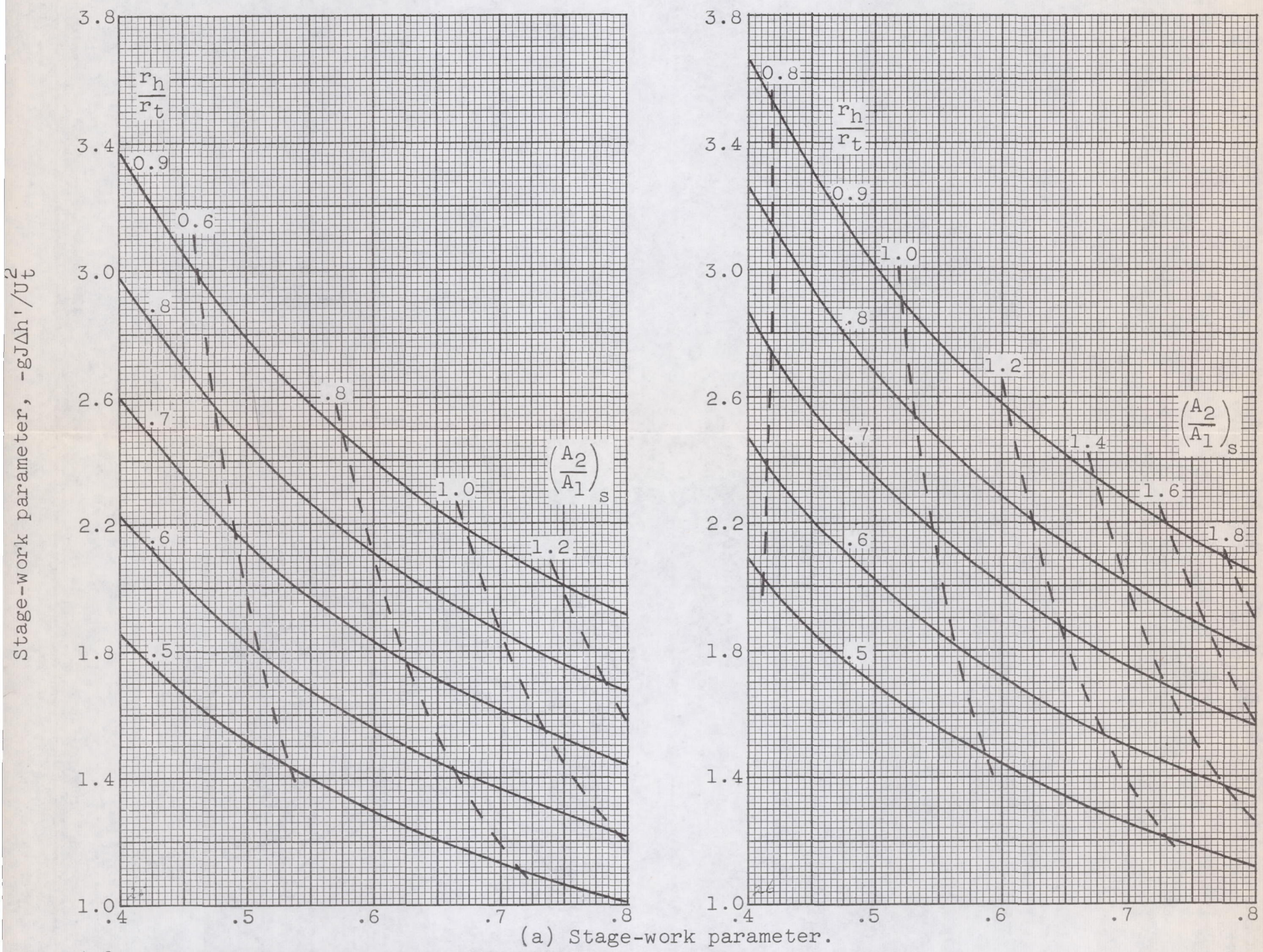
Entrance blade-speed parameter, $U_t/a'_{cr,1}$

2. Diffusion factor, 0.4.

(b) Continued. Ratio of tangential velocities.

Chart I. - Continued. One-stage turbine with one downstream stator.





1. Diffusion factor, 0.4.

2. Diffusion factor, 0.6.

(b) Ratio of tangential velocities.

Chart II. - One-stage turbine with one downstream stator and a rotor relative turning angle of 120° at hub radius.

